**Classical Electrodynamics Qualifying Exam: August, 2012**

1. [10] A thin charged disk of radius $R$ has uniform area charge density $\sigma$. Find the electrostatic potential $\Phi(z)$ along $\hat{z}$, its symmetry axis. Check that your result makes sense when $z \gg R$.

2. [10] A neutral, conducting sphere of radius $a$ is placed in a uniform external electric field $E_0\,\hat{z}$. Charge is induced on the sphere, modifying the external field. Find the electrostatic potential $\Phi(r, \theta)$ outside the sphere. Adopt spherical coordinates with $\hat{z}$ the polar axis.

3. [10] Show that, for a spherically symmetric charge distribution, all multipole moments beyond the monopole vanish. (Hint: Recall that the spherical harmonics are orthogonal.)

4. [20] A paraboloidal surface $z = r_\perp^2/r_0$, extending from $r_\perp = 0$ to $r_\perp = r_0$ ($r_\perp$ and $z$ are cylindrical coordinates), spins with angular velocity $\omega\,\hat{z}$ and carries a surface-charge density

$$\sigma(r_\perp) = \frac{Q}{r_0^2}\left(1 + \frac{4r_\perp^2}{r_0^2}\right)^{-1/2}$$

where $\omega$, $Q$ and $r_0$ are constants.
a) [10] Find the magnetic dipole moment $\vec{m}$.
b) [10] Find the magnetic induction $\vec{B}(r, \theta, \phi)$ in spherical coordinates, in the limit $r \gg r_0$.

5. [10] Starting with Maxwell's equations in vacuum (i.e., $\epsilon = \epsilon_0$ and $\mu = \mu_0$, but free charges and currents are allowed), derive the wave equations satisfied by the scalar and vector potentials in the Lorenz gauge.