Classical Electrodynamics Qualifying Exam: August, 2011

1. [15] Charge q is uniformly distributed around a circular ring of radius a. The ring's axis is the z-axis and its center is located at $z = b$. Find the potential $\Phi(r, \theta)$ in spherical coordinates as a series involving Legendre polynomials in $\cos\theta$ and powers of r.

2. [15] The region above the x-y plane (where $z > 0$) contains a linear isotropic dielectric with dielectric constant ϵ_1/ϵ_0. The region below the x-y plane (where $z < 0$) contains a linear isotropic dielectric with dielectric constant ϵ_2/ϵ_0. A point charge q is located on the z-axis at $z = d$.
a) [10] Find the electrostatic potential $\Phi(r, \phi, z)$ in cylindrical coordinates everywhere in space.
b) [5] Find the bound charge surface density $\sigma_b(r, \phi)$ on the x-y plane.

3. [15] Consider a thick, hemispherical shell of ferromagnetic material. With the z-axis as the polar axis for a spherical coordinate system, the shell occupies $a \leq r \leq b$, $0 \leq \theta \leq \pi/2$, and $0 \leq \phi \leq 2\pi$. The shell has a magnetization $Az\,\hat{z}$, with A a constant. Find the magnetic field $\vec{H}$ at the origin.

4. [15] A point charge q moves with constant velocity $\beta c\hat{z}$ and is at the origin at time $t = 0$. Find the electric field at the origin $\vec{E}(t)$ for time $t > 0$.