Classical Electrodynamics Qualifying Exam: August 24, 2010

1. [30] A neutral conducting sphere with radius a is centered on the origin. A line charge with uniform charge per unit length λ lies on the z-axis between $z = b$ and $z = c$ ($a < b < c$). Find the electrostatic potential $\Phi(r, \theta, \phi)$ for $r > c$ as a series involving $P_l(\cos\theta)$ and powers of r.

2. [20] A dielectric cylinder has length L and radius a. The z-axis is the symmetry axis and the two end faces are at $z = 0$ and $z = L$. The cylinder has a uniform polarization $P\hat{z}$. Find the electric field on the z-axis within the cylinder.

3. [20] A cylinder has length L and radius a and carries a surface-charge density σ on its curved face. (There is no charge on the end faces.) The z-axis is the symmetry axis and the two end faces are at $z = 0$ and $z = L$. The cylinder spins about its axis with angular speed ω. Find the magnetic induction $\vec{B}(z)$ on the z-axis for $z \gg L$.

4. [20] a) Show that a plane electromagnetic wave

$$\vec{E} = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{x} - \omega t)} \qquad ; \qquad \vec{B} = \vec{B}_0\, e^{i(\vec{k}\cdot\vec{x} - \omega t)} \qquad ,$$

with $\vec{E}_0$ and $\vec{B}_0$ constants, satisfies Maxwell's equations in vacuum if $\hat{k} \times \vec{E} = c\vec{B}$; c is the speed of light.
b) Show that $c^2\epsilon_0\mu_0 = 1$.