Qualifying Exam for PHYS 685: Dec 12, 2008

1. [25] A thin spherical shell with radius a is centered on the origin. The "Northern Hemisphere" (i.e., $0 \leq \theta < \pi/2$) carries electric charge area density σ_0 and the "Southern Hemisphere" (i.e., $\pi/2 < \theta \leq \pi$) carries electric charge area density $-\sigma_0$.
a) [15] Find the electrostatic potential $\Phi(r, \theta)$ for $r > a$ as a series involving powers of r and Legendre polynomials. Note: Your answer should include a factor $\int_0^1 du\, P_l(u)$. You need not evaluate this integral.
b) [5] Find the electric dipole moment $\vec{p}$ of the shell.
c) [5] Show that your results in parts (a) and (b) are consistent.

2. [25] A thin disk of radius a lies in the $x - y$ plane with its center at the origin. It carries an electric charge area density

$$\sigma_0 \left(1 - \frac{5}{4}\frac{r}{a}\right)$$

and is spinning about $\hat{z}$ with angular speed ω.
a) [10] Find its magnetic dipole moment $\vec{m}$.
b) [15] Find the magnetic induction $\vec{B}(z)$ along the z-axis for $z \gg a$.

3. [10] Two square metal plates of side length L are separated by a distance d ($d \ll L$). A dielectric slab of size $L \times L \times d$ just slides between the plates. It is inserted a distance x (with one side of the dielectric slab parallel to one side of the metal plates) and held there. The metal plates are then charged to a potential difference V and disconnected from the battery. Find the electric force on the slab.

4. [10] A rectangular loop of wire with non-zero resitance is turned through 180° in a region with static, uniform magnetic induction. Show that the total charge transported through the loop as it is flipped is independent of the speed of flipping.