Classical Mechanics Qualifier (August 2016)
George Mason University

You will have **THREE** hours to complete the exam. Choose **three** out of the following **four** problems. You are allowed to use your graduate textbook during the exam.

Problem 1 (25pts)

a) The Lagrangian of a system is given by

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y} - h\sqrt{x^2+y^2}$$

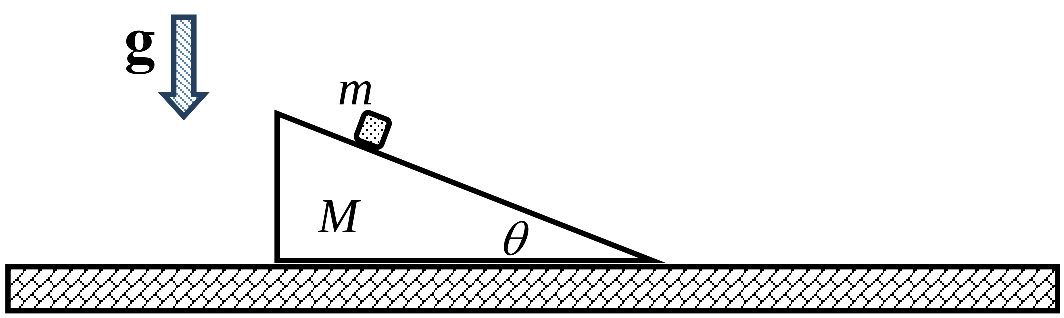
where $a, b, c, f, g,$ and h are constants. What is the Hamiltonian for the system? What quantities are conserved?

b) For the following transformation,

$$Q = q^\alpha \cos(\beta p), \quad P = q^\alpha \sin(\beta p)$$

use the Poisson bracket to determine the values for α and β such the transformation will be canonical.

Problem 2 (25pts)



A small mass *m* slides without friction on a wedge with mass *M* as shown. The inclined side of the wedge makes an angle θ with respect to the table and the wedge can move frictionlessly on the table.

a) Treating the constraint of the small mass staying on the wedge by the method of Lagrange multipliers to obtain the equations of motion for the small mass *m* and the wedge *M*.
b) What are the constants of motion for this system?
c) Find the forces of constraint in keeping the small mass *m* on the wedge.

Problem 3 (25pts)

Two particles with masses m_1 and m_2 attract to each other under a logarithmic potential $U(r) = U_0 \ln(r/a)$ where r is the relative distance between the two masses and $a > 0$.

a) Sketch the effective potential $U_{eff}(r)$ for this central force problem.
b) Find the radius r_0 of all circular orbits as a function of angular momentum l.
c) For small deviations about the circular orbit (i.e., $r(t) = r_0 + \eta(t)$), derive the equation of motion for the deviation $\eta(t)$.
d) By what angle does the apside of the perturbed orbit change during one period of the radial motion?

Problem 4 (25pts)

A physical planar double pendulum is formed by two identical rods of length *l* and mass *m* connected to a horizontal platform and together by small flexible pieces of massless string. Find the frequencies of the normal modes of this system for small oscillations around the equilibrium position. Describe the motion of each of the normal modes.