Classical Mechanics Qualifier (August 2013)
George Mason University

You will have **THREE** hours to complete the exam.
You are allowed to use your graduate textbook during the exam.
Choose **4 out of the 5** problems below.

Problem 1 (25pts)

A partice with mass m and charge q moves in the x-y plane in a conservative potential $V(r)$ and an uniform magnetic field **B** in the z direction with a vector potential given by $\mathbf{A} = \mathbf{B} \times \mathbf{r} / 2$. Use polar coordinates (r, θ) on the plane as your generalized coordinates.

a) Find the Hamiltonian for the particle.

b) Find the two constants of motion for the system.

c) Find the Hamiltonian equations of motion for the sytem. Derive a single 2nd order differential equation of motion in terms of only r.

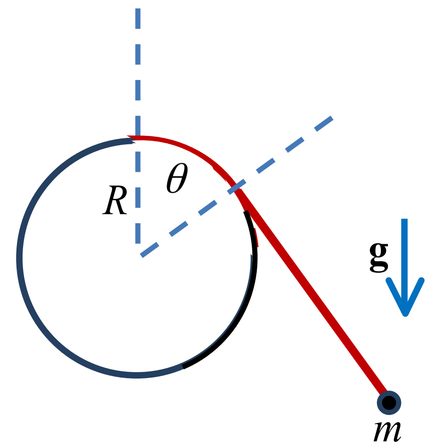
Problem 2 (25pts)

Two particles with masses m_1 and m_2 attract each other under a logarithmic potential $U(r) = U_0 \ln(r/a)$, where r is the distance between the two masses and $a > 0$.

a) Sketch the effective potential $U_{eff}(r)$ for this central force problem.
b) Find the radius r_0 of all circular orbits as a function of the angular momentum l.
c) For small deviations from the circular orbit (i.e., $r(t) = r_0 + \eta(t)$), derive the equation of motion for the deviation $\eta(t)$.
d) By what angle does the apside of the perturbed orbit change during one period of the radial motion?

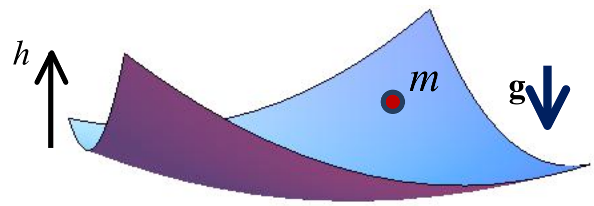
Problem 3 (25pts)

Mass m is attached to the top of fixed disk with radius R by a massless string of length l as shown $(l > R\pi/2)$.



a) Write down the equation of motion for the mass m in terms of the generalized coordinate θ.
b) What is the equilibrium position θ_0 for the motion?
c) Find the frequency of small oscillations around this equilibrium position.

Problem 4 (25pts)



A particle with mass *m* moves under gravity on a smooth surface given by the equation:

$$h(x,y) = x^2 + y^2 - xy$$

a) Using the Lagrangian formalism, find the equations of motion for the particle.

b) Consider small oscillations about the origin. What are the frequencies of the normal modes?

c) If one is to release the mass close to the origin, what must be the ratio of the *x* and *y* displacements so that only the higher frequency normal mode oscillation will be excited?

Problem 5 (25pts)

A large spherical planet with mass M rotates with a constant angular velocity ω_0. Suddenly, an asteroid with mass αM collides radially with the planet at a location given by the colatitude θ (i.e., the latitude is $90^{\circ} - \theta$). Assume that the planet kept its spherical shape after the collision and the size of the asteroid is sufficiently small to be treated as a point mass stuck onto the surface of the planet. Also, assume α to be small.

a) What is the new moment of inertia tensor for the combined object with respect to its new center of mass and along its principle axes? [The moment of inertia for a solid sphere with respect to its center is $I_{sphere} = \frac{2}{5}MR^2$.]

b) Since the combined object is no longer spherically symmetric, the new rotational axis will now rotate in space. What is the period of precession of the rotational axis in terms of the original period of rotation $2\pi / \omega_0$?