Classical Mechanics Qualifier (Fall 2009)
George Mason University

You will have **two** hours to complete all of the following problems.

Short Answers (4 x 5 pts = 20 pts):

S1 (5 pts). The kinetic energy and the potential energy of a spherical pendulum can be written in terms of the generalized coordinates θ and ϕ as:

$$T = \frac{1}{2} m\left(l^2\dot{\theta}^2 + l^2 \sin^2\theta\, \dot{\phi}^2\right)$$

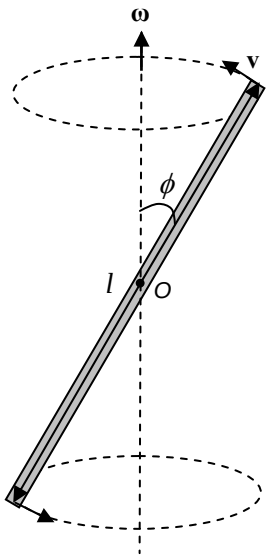
$$U = -mgl\cos\theta$$

where *m* is the mass and *l* is a length of the pendulum. Are either θ and/or ϕ cyclic? What are the conserved quantities for this system?

S2 (5 pts). Planet X is orbiting its Star in a circular orbit. If the Star's mass suddenly decreases by half, what orbit will Planet X now have? Will Planet X still be bounded to the Star?

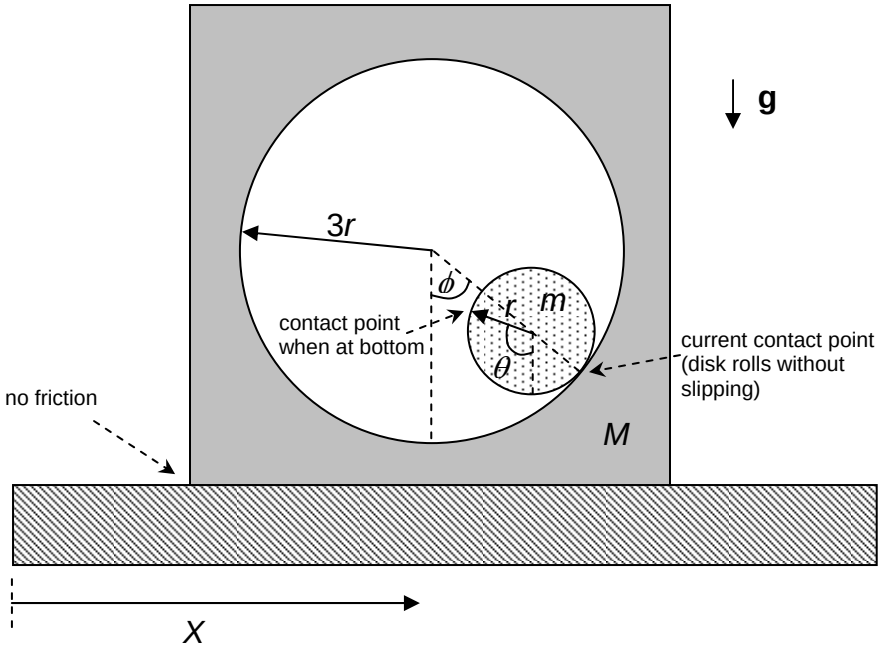
S3 (5 pts). A particle of mass *m* is moving under the influence of a central force given by $f(r) = kr^{\alpha}$ where *k* and α are positive constants. Using the plane polar coordinates (r, θ) as your generalized coordinates, find Hamilton's equations of motion.

S4 (5 pts). A long thin cylindrical rod with length *l* and mass *m* rotates around a fixed axis with frequency ω as shown. Find the torque (in the body axes) with respect to *O* (CM of the rod) required to maintain the motion around $\boldsymbol{\omega}$.



Problem (35 pts):

A disk of mass m and radius r rolls without slipping inside a circular opening, of radius $3r$, within a block of mass M. The block slides without friction on a horizontal surface. (Take $U = 0$ when the disk is at the bottom of the well.)



a) (5 pts) Write down the Lagrangian for this system using the generalized coordinates (X, ϕ, and θ) indicated in the illustration above.
b) (5 pts) Write down the constraint condition for the disk rolling without slipping inside the circular opening.
c) (10 pts) Obtain the equation of motion for the generalized coordinates (X and ϕ).
d) (10 pts) Assuming small angular deviations and $\dot{X}$ and $\dot{\phi}$ to be small, find the frequency of small oscillations of the disk inside the block.
e) (5 pts)If M is not allowed to move, what will the frequency of small oscillations be?