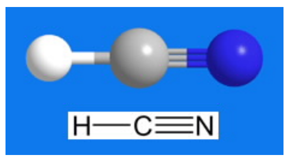
Qualifying exam - January 2017

Statistical Mechanics

You can use one textbook. Please write legibly and show all steps of your derivations.

Problem 1 [20 points]
Calculate the internal energy (in J/mole) and specific heat at a constant volume (in J/mole/K) of hydrogen cyanide HCN at the temperature of 800 K. Consider HCN as an ideal gas and treat the molecular rotations and vibrations in the classical limit. The HCN molecule has a linear structure H−C≡N (see figure below). The gas constant is $R = 8.314$ J/mole/K.



Problem 2 [30 points]
Imagine a harmonic solid with an isotropic dispersion relation $\omega = Ak^b$, where ω is the angular frequency of atomic vibrations, k is the wave number, and $A > 0$ and $b > 0$ are constants. Assuming that this dispersion relation holds for all three polarizations of phonons, show that in the low-temperature limit the phonon contribution to the heat capacity of the solid is proportional to $T^{3/b}$.

Problem 3 [25 points]
Consider a cavity containing black-body radiation at a temperature T_1. Suppose the volume of the cavity increases in an equilibrium adiabatic process from an initial value V_1 to a final value $V_2 = 8V_1$.

1. What is the final temperature T_2 in the cavity? [5 points]
2. If the initial radiation pressure was p_1, what is the final pressure p_2? [5 points]
3. If the cavity initially contained a total of N_1 photons, what is the final number N_2 of photons in the cavity? Explain the physical meaning of this result. [15 points]

Problem 4 [25 points]
Consider a three-dimensional free electron gas at zero temperature (degenerate electron gas). Calculate the relative root-mean-square deviation of its energy,

$$\frac{\left(\overline{(\varepsilon - \overline{\varepsilon})^2}\right)^{1/2}}{\overline{\varepsilon}}, \tag{1}$$

where ε is energy per electron.