QM Qualifier: August 2016 Name_____

You are allowed to use one textbook of your choice.

Problem 1 (25pts)

A spin-1/2 particle is initially in the eigenstate $|x+\rangle$ of S_x with the eigenvalue $\frac{\hbar}{2}$. A magnetic field of Larmor frequency ω is turned on at time $t=0$ in the *x*-*z* plane, making an angle θ with the *z*-axis.

(a). (10pts) Find the state vector $|\alpha(t)\rangle$ at any given time *t*.

(b). (10pts) Is $\langle x+|\alpha(t)\rangle$ a periodic function of time? If yes, what is the period T ?

(c). (5pts) Is $|\langle x+|\alpha(t)\rangle|$ a periodic function of time? If yes, what is the period T' ?

Problem 2 (20pts)

A one-dimensional simple harmonic oscillator of angular frequency ω and mass m is in a number state $|n\rangle$.

(a) (10pts) Calculate the uncertainties of position and momentum in this state.

(b) (5pts) Prove that the uncertainty principle is satisfied in this state.

(c) (5pts) What is the energy of the minimum uncertainty state?

Problem 3 (25pts)

Density matrices

$\rho_1 = \frac{1}{4}\begin{pmatrix} 3 & -i \\ i & 1 \end{pmatrix}$ and $\rho_2 = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\frac{\pi}{4}} \\ e^{i\frac{\pi}{4}} & 1 \end{pmatrix}$ represent two states of an ensemble of particles .

(a) (5pts) Identify which of these states are pure or mixed.

(b) (10pts) Find the state vector represented by the density matrix of each pure state.

(c) (10pts) Apply a rotation about *z*-axis by an angle β . Calculate both density matrices after the rotation.

Problem 4 (30 pts)

A system consists of two different spin 1/2 particles. Let $\vec{S}_1$ and $\vec{S}_2$ be the individual spin operators and $\vec{S} = \vec{S}_1 + \vec{S}_2$ the total spin operator. The spin-spin coupling Hamiltonian is $H = \gamma \vec{S}_1 \cdot \vec{S}_2$, where γ is a real constant.

(a). (10pts) Find the eigenstates and eigenvalues of H .

(b). (10pts) Which of these eigenstates has overall zero spin in the z-direction, $\langle S_z \rangle = 0$?

(c). (10pts) Pick a state with total $\langle S_z \rangle = 0$ and measure the spin projection S_{1z} of the first particle. What are the possible measured values, corresponding probabilities, and the post--measurement states of the full system?