NAME:

Quantum Mechanics, Qualifying Exam, Aug. 2013

Note: If a formula appears in the book, please use that as a starting point, there is no need to show the derivation of that formula.

(1) Obtain the energy eigenvalues and eigenfunctions of a two-dimensional quantum mechanical rigid rotor of moment of inertia I. Does the system have zero point energy? Explain your answer. [15 pts]

(2) Consider a particle of mass m in one dimension, bound to a fixed point by a δ-function potential, $V(x) = -a\delta(x)$, where a is real and positive. Find the normalized wave function and binding energy of the ground state. Are there excited bound states? [15 pts]

(3) (a) A hydrogen atom is in a spherically symmetric state. What is the most general wave function for such state?
(b) Show that for the ground state of the hydrogen atom, the most probable radius of the electron is the Bohr radius.
(c) What spherically symmetric eigenstate of the hydrogen atom has a most probable radius of 4 times the Bohr radius? Show your steps. [15 pts]

(4) At $t = 0$, a particle of mass m in one dimension confined by potential $V(x) = 2(x-1)^2$ is equally likely to be in the ground and the first excited state. What is the wave function of

the system at $t = 0$? What is the wave function of the particle at a later time $t > 0$? If the particle is also subject to a constant electric field B along x direction, how does your answers change? [20 pts]

(5) A particle in a spherically symmetric potential is described by the following wave functions,
(a) $\psi(x, y, z) = xy$,
(b) $\psi(x, y, z) = x + y + z$.
In each case, calculate the possible angular momentum quantum numbers and the probability of finding the particle in each of these angular momentum eigenstates. [20 pts]

(6) Consider a particle of mass m and energy E moving in three dimensions. What is the most general wave function of the particle
(a) if it is an eigenstate of energy.
(b) if it is an eigenstate of momentum.
(c) if it is an eigenstate of parity.
(d) if it has zero average momentum.
(e) if it is an eigenstate of angular momentum with quantum number $l = 0$. [15 pts]