Qualifier

1. (25 points) A static magnetic field is applied to a spin-1/2 particle in the *z*-direction. The corresponding Hamiltonian is $H = \omega S_z$, where ω denotes the Larmor frequency.

(a) (10 points) At time $t = 0$, the particle starts from the eigenstate of operator $\vec{S}\cdot\hat{n}$ with a positive eigenvalue, where the unit vector $\hat{n}$ lies in the *x*-*z* plane with a polar angle θ. Find the matrix form of the state vector at time *t* in the S_z eigenbasis.

(b) (5 points) If the system consists of a pure ensemble of spin-1/2 particles, starting from the initial same state as in (a), find the density matrix at time *t*.

(c) (10 points) If the system is a mixed ensemble, 50% of the particle's population starts from the same initial state as in (a) and the rest starts from the eigenstate of $\vec{S}\cdot\hat{n}$ with the negative eigenvalue. What should be the density matrix at any given time?

2. (25 points) Let $|nlm\rangle$ represents the energy eigenstate for an isolated hydrogen atom, neglecting the electron spin. n is the principal quantum number, l and m denote the eigenket of the orbital angular momentum. For state $|\alpha\rangle = \frac{1}{3}(|210\rangle + \sqrt{3}|211\rangle + \sqrt{5}|21-1\rangle)$ answer the following questions.

(a) (10 points) Calculate the expectation values $\langle L_x \rangle$, $\langle L_y \rangle$, and $\langle L_z \rangle$.

(b) (10 points) Calculate the expectation values $\langle L_x^2 \rangle$, $\langle L_y^2 \rangle$, and $\langle L_z^2 \rangle$.

(c) (5 points) Verify the uncertainty principle for all three components of orbital angular momentum.

3. (25 points) A one dimensional simple harmonic oscillator with an angular frequency ω is in a coherent state $|\lambda\rangle$ at time t = 0. Here $|\lambda\rangle$ is the eigenket of the annihilation operator, $a|\lambda\rangle = \lambda|\lambda\rangle$.

(a) (10 points) Calculate the expectation value of position and momentum at time *t*.

(b) (10 points) Calculate the uncertainty for position and momentum at time *t*.

(c) (5 points) Prove the coherent state is a minimum uncertainty state.

4. (15 points) A system of two spin-1/2 particles is in a spin-singlet state, $|\alpha\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$. Observer Alice can only make measurement on one of the particles. Observer Bob can only measure the other.

(a) (5 points) When Alice measures S_z, what are the possible values she should get if Bob does not measure his particle? What are the corresponding probabilities?

(b) (5 points) What results should Alice get if Bob measures his particle at the same time and gets $-\frac{\hbar}{2}$?

(c) (5 points) Assuming a source that can generate such particle-pairs one at a time, design a thought experiment so that the results break the Bell's inequality. List your measurements and corresponding results. Show that the Bell's inequality is violated by the results.