NAME:

PhD Qualifying Exam, 2019 (Open book -Sakurai)

(1) Consider a particle of mass m subjected to a one-dimensional potential of the following form,

$$\begin{aligned} V(x) &= \frac{1}{2}kx^2, \quad x \geq 0 \\ &= \infty, \quad x < 0 \end{aligned}$$

(a) What are the eigenvalues and the eigenfunctions of the particle ? Explain your answer.
(b) What is the expectation value $< x^2 >$ for the ground state.
(c) At $t = 0$, particle is equally likely to be in the ground and the first excited state. What is the wave function of the particle at $t = 1$ sec..
(d) In the region $x \geq 0$, If the particle is also subjected to a constant force F, how does the answers to the above parts (a-c) change ?.
(e) Write the unitary operator that relates the wave function with $F = 0$ to the wave function with $F \neq 0$.

(2) Consider a free particle of mass m, charge e and energy E moving in three dimension. What is the most general wave function of the particle
(a)if it is an eigenstate of energy.
(b) If the eigenstate is also an eigenstate of momentum.
(c) If the eigenstate is also an eigenstate of parity.
(d) If the eigenstate has zero average linear momentum.
(e) If the particle is in the eigenstate of angular momentum quantum number $l = 1$.
(f) Does the energy of the particle depend upon the angular momentum quantum number l?. Explain
(h) In each of the above cases (a-e), how does the wave function change under time reversal ?.
(g) If the particle is subjected to a magnetic field $\vec{B}$, write down the Hamiltonian of the system.

(3) Consider a particle in a potential $V(r) = A/r^a$, where a is a real number.
(a) If the particle is in the $l = 1$ angular momentum state, write down the the most general angular part of the wave function.
(b) Do you expect the eigenvalues to depend upon the angular momentum quantum numbers, stating clearly how your answer depends upon a.
(c) Write down the most general angular parts of the wave functions that are also the eigenstates of the parity.
(d)Explain why the above wave functions are in general complex while the one-dimensional harmonic oscillator wave functions are real.
(e) For $a = 1$ and $a = -2$, write down the ground state wave functions of the particle and calculate the most probable values of r where the particle will be found.
(f) If a particle is described by a wave function $\psi(x, y, z) = Ne^{-2r^2}(x + z)$ where N is a normalization constant, calculate the possible angular momentum quantum numbers (l, m) of the system.

(4) Consider a system described by the Hamiltonian H,

$$H = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix},$$

where a is a constant.
(a) At $t = 0$, we measure the energy of the system, what possible values will we obtain?
(b) At later time t, we measure the energy again, how is it related to its value we obtain at $t = 0$?
(c) If at $t = 0$, the system is equally likely to be in its two possible states, write down the most general state of the system at $t = 0$.
(d) What is the probability that at time $t = 5$, the system will be in a state different from its initial state?.
(e) Suppose the above Hamiltonian describes a spin-1/2 particle in a magnetic field. If S_x is found to be $\hbar/2$, what is the probability of getting S_z equal to $\hbar/2$?. What is the probability of getting S_y equal to $-\hbar/2$? What is the probability of getting S_x equal to $-\hbar/2$