NAME:

QM, Qualifying Exam: 2012

Note: This is an open book exam and you are allowed to bring Sakurai or Shankar's book on quantum mechanics. If a formula appears in the book, please use that as a starting point, there is no need to show the derivation of that formula.

(I) (10 points) Consider a particle of mass m, at absolute zero temperature, confined to a rectangular box of sides $(a, a, a/2)$.
(a)(5 pts) Using the uncertainty principle, derive the energy of the particle.
(b)(2 pts)How does your answer compare with the exact calculation?
(c)(3 pts)For the exact calculation, please give the first two energy levels and the degeneracy associated with each level.

(II) (20 points) Consider a particle of unit mass in a potential $V(x, y) = A\{x^2 + (y - 1)^2\}$, $A > 0$.
(a)(11 pts) Write down for the ground and first excited state: (1) energy (2) degeneracy and (3) the wavefunction.
(b)(6 pts)What is the expectation value of position and momentum of the particle in the ground state?
(c)(3 pts)If at an initial time t_0, the particle is equally likely to be in the first and second excited state, what is the wave function at some later time t.

(III) (10 points) At a given instance in time, a free particle of mass m is described by a Gaussian wave packet of width unity. The average position of the particle is 0.5 and average momentum is p_0. In this problem you need not worry about normalization.
(a)(3 pts) What is the wave function of the particle?
(b)(4 pts) Is the wave function an eigenfunction of (1) position (2) momentum (3) energy? Explain why or why not.
(c)(3 pts) Under what condition does this wave function describes an eigenfunction of a harmonic oscillator?

(IV) (20 points) Consider a particle in a potential $V(r) = 1/(r^2 + 1)$. The particle is in an eigenfunction of the Hamiltonian such that the square of the net angular momentum is $20\hbar^2$.
(a)(5 pts) What is the most general angular part of the wave function?
(b) If in addition it is in an eigenstate of L_z with quantum number $m = 2$ calculate:
1)(4 pts)the expectation value of $L_x^2 + L_y^2$.
2)(5 pts)the expectation value of $L_x L_y$. Is $L_x L_y$ an observable?
3)(6 pts)the expectation value of p_x and p_y. Hint: What is the commutation relation between p_x and p_y with L_z?

(V) (20 points) Suppose a particle has the wave function, $\psi(x, y, z) = A(r)[1 + (2x + z)/r]$, where $A(r)$ is a radial wave function chosen so that ψ is normalized.

(a)(5 pts)What are the possible angular momentum quantum numbers of the system?
(b)(5 pts)What are the probabilities for the different possible outcomes of the measurement of L_z?
(c)(5 pts)What is the expectation value of L_z?
(d)(5 pts)What is the rms uncertainty in L_z?