Qualifier Aug. 2015 name______________________________

You are allowed to bring one textbook of your choice.

Problem 1 (20points)

A large number N of spin ½ particles are prepared in the same eigenstate of the operator $\hat{S}_x$. This ensemble goes through a series of Stern-Gerlach-type of measurements as follows:

1) The first measurement lets $s_z = -\hbar/2$ particles pass, and blocks $s_z = \hbar/2$ particles.
2) The second measurement blocks $s_n = -\hbar/2$ and let $s_n = \hbar/2$ pass, where s_n is the eigenvalue of the operator $\vec{S} \cdot \hat{n}$ and the unit vector $\hat{n}$ makes an angle θ from the z-axis in the z-x plane.
3) The final step measures $\hat{S}_x$.

Calculate the number of particles (N_+) with $s_x = \hbar/2$ and (N_-) with $s_x = -\hbar/2$, respectively, in the final beam after the third step. In which quadrant should the θ value be to maximize N_+?

Problem 2 (30points)

An electron is subject to a uniform magnetic field along the z-axis. The Larmor frequency is ω . At t=0 the electron is in the eigenstate of $\vec{S} \cdot \hat{n}$ with $s_n = \hbar/2$, where the unit vector $\hat{n}$ makes an angle θ from the z-axis in the z-x plane.

(a) Calculate the expectation values of $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ at time t.
(b) Calculate the uncertainty of all three observables from the part (a).
(c) Show an example that the uncertainty principle is satisfied at time t.

Problem 3 (20 points)

A system consists of two spin ½ particles localized at the same point in space. Let $\vec{S}_1$ and $\vec{S}_2$ be the individual spin operators of the two particles respectively. The spin-spin coupling Hamiltonian of this system is $H = S_{1x}S_{2x} + S_{1y}S_{2y} + S_{1z}S_{2z}$. Find the eigenstates and the corresponding eigenvalues of H in the following cases:

(a) if the two particles are different (for example, an electron and a proton)
(b) if the two particles are identical.

Problem 4 (30points)

The state $|\alpha\rangle$ of a simple harmonic oscillator at t=0 is a superposition of two number states $|n\rangle$ and $|l\rangle$, where $n > l$.

(a) Construct $|\alpha\rangle$ so that the expectation value of position operator $\langle X \rangle$ is at its maximum for a given n.

(b) Calculate the variance of the position operator, $\langle (X - \langle X \rangle)^2 \rangle$ at time t.

(c) Calculate the expectation value of the number operator N at time t.