

## Qualifying Exam, Electromagnetic Theory, Aug. 2013

1. [20 pts] Charge  $q$  is uniformly distributed on a straight line of length  $l$  along the  $z$  direction. Choose the middle point of the line charge as the origin.
  - (a) Compute the electric dipole and quadrupole moments of the line charge.
  - (b) Find the scalar potential  $\Phi(r, \theta, \phi)$  for  $r \gg l$  to the order of  $1/r^3$ .
  
2. [20 pts] The top surface of a cubic box is maintained at constant potential  $V$ , while all other surfaces are grounded at potential zero. The side length of the box is  $a$ .
  - (a) Write down the differential equation for the potential  $\Phi(x, y, z)$  inside the cube and its boundary conditions in cartesian coordinate system.
  - (b) Solve for  $\Phi(x, y, z)$ . You may express the answer as a series sum.
  
3. [30 pts] A spherical thin shell of radius  $R$  is made of ideal conductor and initially charge neutral. Then it is placed in a uniform electric field  $E_0 \hat{z}$ .
  - (a) Find the scalar potential  $\Phi$  outside the shell.
  - (b) Find the surface charge density  $\sigma(\theta)$  at the outer surface of the shell, where  $\theta$  is the polar angle defined respect to  $\hat{z}$ .
  - (c) Suppose the shell now cracks into two equal halves along the plane perpendicular to  $\hat{z}$  and going through the center of the shell. How much external force is required to prevent the two halves from separating from each other?
  
4. [30 pts] A circular loop of radius  $R$  carries current  $I$ . It is within the  $xy$  plane and centered at the origin. The current flows counterclockwise when viewed from above the  $xy$  plane.
  - (a) Find the magnetic induction  $\mathbf{B}(z)$  on the  $z$  axis.
  - (b) Find the vector potential  $\mathbf{A}(\mathbf{r})$  off axis for  $r \gg R$ , show your steps.
  - (c) Suppose the current decays over time,  $I(t) = I_0 e^{-t/\tau}$ . In what direction does the electromagnetic energy flow near the origin?