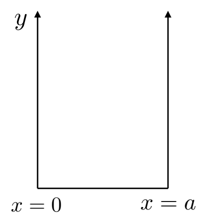
# Qualifying Exam, Electromagnetism, Jan. 2014

1. [30 pts] Charge $q$ is evenly distributed on a ring of radius $a$. The ring is centered at the origin and lies within the $xy$ plane. Assume a vacuum elsewhere.
(a) Show the potential $\Phi$ along the $z$-axis is given by $\Phi(z) = q/4\pi\epsilon_0\sqrt{z^2 + a^2}$.
(b) Write down the form of $\Phi(r, \theta, \phi)$ away from the ring as a series summation, for $r > a$ and $r < a$ respectively.
(c) Determine $\Phi(r, \theta, \phi)$ for $r > a$ from its $z$-axis value obtained in (a).
(d) Compute the electric dipole moment $\mathbf{p}$ and quadrupole moment $Q_{ij}$ of the charged ring, and use them to find $\Phi(r, \theta, \phi)$ for $r \gg a$ to the order of $r^{-3}$. (You may use the multipole moments $q_{lm}$ instead of $\mathbf{p}$ and $Q_{ij}$.)

2. [20 pts] An infinitely long and deep slot is formed by two grounded conductor plates at $x = 0$ and $x = a$, and a conductor plate at $y = 0$ is held at a potential $V$ and insulated from the other plates.
(a) Write down the differential equation for the potential $\Phi(x, y, z)$ inside the slot and its boundary conditions.
(b) Find the potential $\Phi$ everywhere inside the slot.




3. [30 pts] Total charge $Q$ is uniformly distributed on a spherical surface of radius $R$. The sphere is centered at the origin and spins around the $z$ axis with angular velocity $\omega$. Assume magnetic permeability $\mu_0$ everywhere.
(a) Show that away from the spherical surface, the magnetic field can be expressed as $\mathbf{H} = -\nabla\Phi_M$, with $\nabla^2\Phi_M = 0$.
(b) Find the surface current density $\mathbf{K}$ and the boundary conditions for the magnetic induction $\mathbf{B}$ at $r = R$.
(c) Find $\Phi_M$ inside and outside of the sphere.
(d) Find $\mathbf{B}$ inside the sphere.

4. [20 pts] A steady current $I$ carried by a wire along the $z$ axis flows from $z = -\infty$ and abruptly terminates at the origin $z = 0$. The charge accumulated at the origin is $Q(t) = It$, where $t$ is time.
a) Find the electric field $\mathbf{E}(\mathbf{r}, t)$ and the displacement current $\partial_t\mathbf{D}$. Explain your reasoning.
b) Discuss qualitatively how the magnetic field $\mathbf{H}(\mathbf{r})$ for $z > 0$ can be determined from the Ampere-Maxwell law.