Qualifying exam - January 2026

Classical Electrodynamics

You can use one textbook. Please write legibly and show all steps of your derivations. Note the Formula Sheet attached.

Problem 1 [30 points]
A thin non-conducting spherical shell of radius R has the potential

$$V(\theta) = V_0(1 + 5\cos\theta + 6\sin^2\theta)$$

on its surface (θ is the polar angle). Find the potential $\varphi(r, \theta)$ inside and outside the shell.

Problem 2 [25 points]
Consider two concentrical shells with radii R_1 and $R_2 > R_1$. The space between the shells is filled with a dielectric material uniformly polarized with a radial polarization $\mathbf{P} = P\hat{\mathbf{r}}$. Calculate the electric field $\mathbf{E}(\mathbf{r})$ and potential everywhere.

Problem 3 [15 points]
Find the magnetic dipole moment of a thin disk of radius R carrying a uniform surface charge density σ and rotating around its axis (which is perpendicular to the plane of the disk) with an angular velocity ω.

Problem 4 [30 points]
An infinitely long thick wire of radius R is made of an isotropic linear magnetic material with magnetic permeability μ. The wire carries a current I with a current density distributed uniformly over the cross-section.

1. Find the magnetic field $\mathbf{B}$ as a function of the radial distance r from the axis of the wire.
2. Find the magnetization of the material $\mathbf{M}$ as a function of r.
3. Find the volume bound current density $\mathbf{J}_b$.
4. Find the surface bound current density $\mathbf{K}_b$.
5. Find the total bound current in the wire.

Formula Sheet

$$\varphi(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta).$$

$$P_0(x) = 1; \quad P_1(x) = x; \quad P_2(x) = \frac{3x^2 - 1}{2}$$

Orthonormality condition for Legendre polynomials:

$$\frac{2l+1}{2} \int_0^{\pi} P_l(\cos\theta) P_{l'}(\cos\theta) \sin\theta d\theta = \delta_{ll'}.$$