Qualifying exam - January 2025

Classical Electrodynamics

You can use one textbook. Please write legibly and show all steps of your derivations. Note the Formula Sheet attached.

Problem 1 [40 points]
Consider an infinitely thin disk of radius R carrying a uniformly distributed charge q.

1. [10 points] Using the azimuthal symmetry, calculate the potential at any point on the symmetry axis z of the disk.

2. [30 points] Calculate the potential at any point $\mathbf{r}$ ($r > R$) as an expansion in Legendre polynomials $P_n(\cos\theta)$, where θ is the polar angle (Fig. 1).

Note the Taylor expansion in the Formula Sheet.

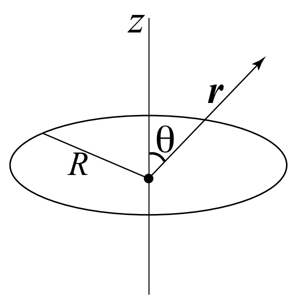

Figure 1: Polar coordinates for a uniformly charged disk.

Problem 2 [40 points]
Calculate the potential energy of a point charge q in empty space a distance x away from a semi-infinite linear dielectric medium with a dielectric constant ε (Fig. 2).

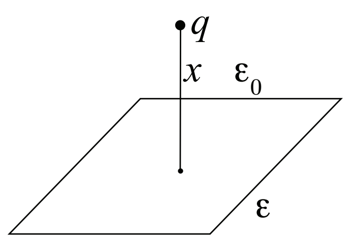

Figure 2: Point charge q a distance x away from a semi-infinite dielectric medium.

Problem 3 [20 points]
A coaxial cable consists of a thin wire carrying current I and a thin coaxial cylindrical shell of radius R carrying an equal net current I in the opposite direction (Fig. 3a). The lower half of the space between the conductors is filled with an isotropic linear magnetic material with a permeability μ, while the upper half remains empty space.

1. [10 points] Calculate the magnetic fields $\mathbf{B}$ and $\mathbf{H}$ everywhere.

2. [10 points] Calculate the magnetization $\mathbf{M}$ of the magnetic material.

Hint: Despite the presence of the magnetic material, the field lines in the magnetic material ($\mathbf{B}_m$ and $\mathbf{H}_m$) and in empty space ($\mathbf{B}_0$ and $\mathbf{H}_0$) circulate around the axis of the cable, as shown in (Fig. 3b).

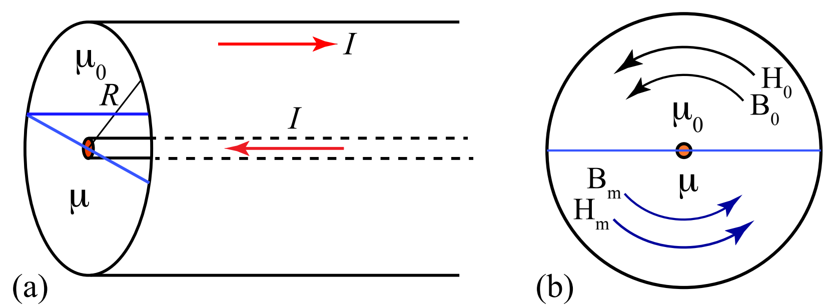

Figure 3: (a) Coaxial cable whose lower half is filled with magnetic material. (b) Cross-sectional view of the cable showing the directions of the fields in the magnetic material ($\mathbf{B}_m$ and $\mathbf{H}_m$) and in empty space above it ($\mathbf{B}_0$ and $\mathbf{H}_0$).

Formula Sheet

$$\int \frac{x dx}{(x^2+a)^{1/2}} = (x^2+a)^{1/2}.$$

$$\int \frac{x dx}{(x^2+a)^3} = -\frac{1}{4(x^2+a)^2}.$$

Note the Taylor expansion:

$$(1+x)^{1/2} = 1 + \sum_{k=1}^{\infty} \binom{1/2}{k} x^k.$$

You do not have to specify the binomial coefficients $\binom{1/2}{k}$. Simply keep them in this form in the solution.