Qualifying exam - January 2024

Classical Electrodynamics

You can use one textbook. Please write legibly and show all steps of your derivations. Note the Formula Sheet attached.

Problem 1 [25 points]
Consider two non-overlaping, spehercally symmetric charge distributions $\rho_1(r)$ and $\rho_2(r')$ centered at points separated by a distance R (Figure 1). Calculate the force between these distributions.

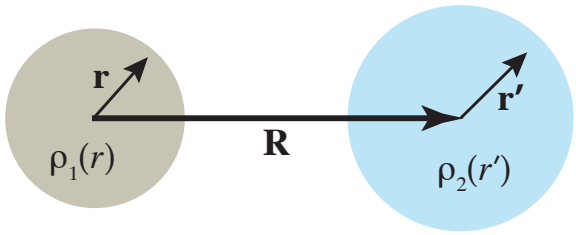

Figure 1: Two non-overlapping spherical charge distributions.

Problem 2 [25 points]
A spherical cavity of a radius R is carved in a unform linear dielectric medium with a permittivity ε. Capacitor plates at infinity produce a fixed uniform electric field $\mathbf{E}_0$. Calculate the electric field inside ($r < R$) and outside ($r > R$) the cavity .

Problem 3 [15 points]
A sphere rotating around its axis with angular velocity ω has a uniform mass density and a uniform surface charge density. The sphere has a total mass M and a net charge Q. Find its gyromagnetic ratio $G = m/L$, where m is the magnetic moment of the sphere and L is its mechanical angular momentum.

Problem 4 [35 points]
An infinitely long thick wire of radius R is made of an isotropic linear magnetic material with magnetic permeability μ (Fig. 2). The wire carries a current I with a current density distributed uniformly over the cross-section.

1. Find the magnetic field $\mathbf{B}$ as a function of the radial distance r from the axis of the wire.

2. Find the magnetization of the material $\mathbf{M}$ as a function of r.

3. Find the volume bound current density $\mathbf{J}_b$.

4. Find the surface bound current density $\mathbf{K}_b$.

5. Find the total bound curent in the wire.

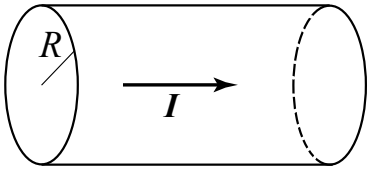

Figure 2: A thick wire made of a linear magnetic material carrying a uniform current I.

Formula Sheet

$$\int \sin^3\theta d\theta = \frac{\cos^3\theta}{3} - \cos\theta.$$