Classical Electrodynamics Qualifying Exam (3 hours)
January 14, 2020

Solving five of the following six problems (Open-book, closed-notes). Passing score: 50 pts.

Q1. [20 points] Two concentric spherical conductors of radii a and $b>a$ and a point charge q between them at a location r such that $a<r<b$. Assuming the spheres are grounded, find the total induced charge on each sphere. (Hint: One possible solution is to use Green's reciprocation theorem.)

Q2 [20 points]. A line charge of length $2d$ with a total charge Q has a linear charge density varying as (d^2-z^2), where z is the distance from the midpoint. A grounded, conducting spherical shell of inner radius $b>d$ is centered at the midpoint of the line charge. (a) Find the potential everywhere inside the spherical shell as an expansion in Legendre polynomials. (b) Calculate the surface-charge density induced on the shell. (c) Discuss your answers to parts a and b in the limit that $d\ll b$.

Q3 [20 pts]. A point charge q is located in free space a distance d from the center of a dielectric sphere of radius a ($a<d$) and dielectric constant ϵ/ϵ_0 . Find the potential at all points in space as an expansion in spherical harmonics.

Q4. [20 pts]. An infinitely long, hollow, rectangular pipe runs along the z-axis and extends from $x=0$ to $x=a$ and $y=0$ to $y=b$. The face at $x=a$ is held at constant potential V , while the remaining three faces are maintained at potential zero. Find the potential Φ(x, y, z) inside the pipe.

Q5 [20 pts]. A conducting sphere with radius R is placed in a uniform electrical field E_0 . Knowing that the potential on the sphere is V_0 , find the electric field $\vec{E}$ outside the sphere and the total charge on the sphere.

Q6 [20 pts]. Neutron stars are formed when a massive star runs out of fuel and collapses. The very central region of the star – the core – collapses, crushing together every proton and electron into a neutron. In a typical neutron star, the magnetic field is trillions of times that of the Earth's magnetic field. Suppose a neutron star consists of close-packed neutrons, and its magnetic field comes from the the magnetic moments of the neutrons. The radius of the neutron is $a=8\times10^{-16}$ m, and the magnetic moment of each neutron is $\mu=9.65\times10^{27}\,\mathrm{A\cdot m^2}$. The magnetic moments of the neutrons are aligned toward the same direction. Estimate the maximum magnetic induction field **B** of the neutron star.