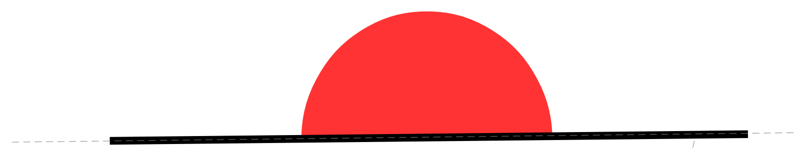
Classical Electrodynamics Qualifying Exam (3 hours)
January 17, 2018
Open-book, closed-notes

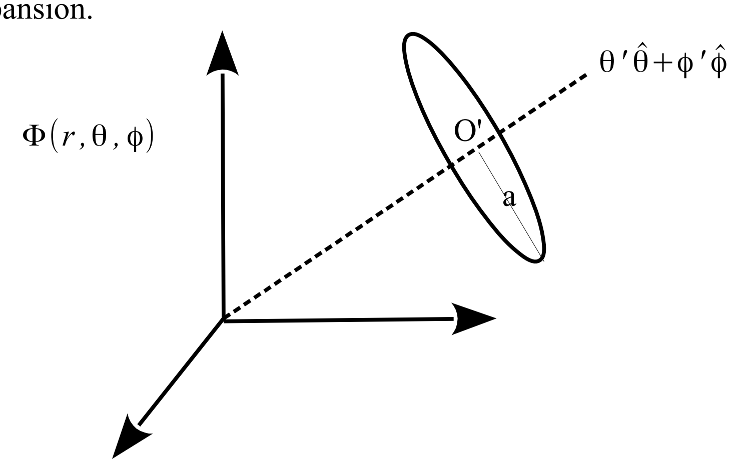
1. [20 pts] Q1. The potential on an isolated spherical shell is not equal anywhere. It is vacuum inside and outside the spherical shell. Prove that the potential at the center of the sphere is the average of the potential on the spherical shell.

2. [20 pts] A solid conducting hemisphere with radius R sits on the ground due to gravity. The density of the hemisphere is ρ . Now, in order to lift the hemisphere from the ground (i.e., the surface of the earth, which is conducting), let's apply a uniform electric field $\vec{E}_0$ whose direction is perpendicular to the ground. How big is E_0 so that the hemisphere can be lifted up? (The acceleration of gravity is g)



3. [20 pts] A point dipole $\vec{p}$ is embedded at the center of a dielectric sphere (radius R and dielectric constant ϵ_r). Find the potential inside and outside of the sphere.

4. [20 pts] In a spherical coordinate system, the axis of a charged ring has the direction $\theta'\hat{\theta}+\phi'\hat{\phi}$, and the center of the ring O' is located at (d,θ',ϕ') . The radius of the ring is a, and charge q is uniformly distributed on the ring. Write the scalar potential $\Phi(r,\theta,\phi)$ anywhere using spherical harmonics expansion.



5. [20 pts] Charge Q is uniformly distributed on a conduction sphere with radius R . The sphere is rotating around its z axis at an angular velocity ω . Suppose the magnetic ϵ permeability μ_0 is the same outside and inside the sphere. Solve for the magnetic induction $\mathbf{B}(r)$ using the known quantities Q , R , and ω for both $r>R$ and $r<R$.