Qualifying exam - August 2024

Classical Electrodynamics

You can use one textbook. Please write legibly and show all steps of your derivations. Note the Formula Sheet attached.

Problem 1 [25 points]
Consider two concentric shells with radii R_1 and $R_2 > R_1$. The space between the shells is filled with a dielectric material uniformly polarized with a polarization $\mathbf{P}$. Calculate the electric field $\mathbf{E}(\mathbf{r})$ everywhere.

Problem 2 [30 points]
The space between two concentric conducting spherical shells with radii R_1 and $R_2 > R_1$ is filled with isotropic linear dielectric material with the dielectric constants ε_{r1} at $R_1 < r < R$ and ε_{r2} at $R < r < R_2$, where $R_1 < R < R_2$. The outer shell (R_2) is grounded and the inner shell (R_1) has a potential φ_0. Calculate the electric field everywhere.

Problem 3 [20 points]
A steady current I flows down an infinitely long cylindrical wire of radius R. The current density $\mathbf{J}(r)$ is distributed over the cross-section of the wire non-uniformly. Knowing that the $\mathbf{H}$-field inside the wire ($r < R$) is

$$\mathbf{H} = \frac{Ir}{2\pi R^3}\hat{\mathbf{J}} \times \mathbf{r},$$

find current density $\mathbf{J}(r)$. Also find the $\mathbf{H}$-field outside the wire.

Problem 4 [25 points]
A thin nonconducting circular disk of radius R carries a uniformly distributed electric charge Q. The disk rotates with angular velocity ω around the axis perpendicular to its plane through its center (Fig. 1). Calculate the magnetic field on the axis a distance z from the center of the disk.

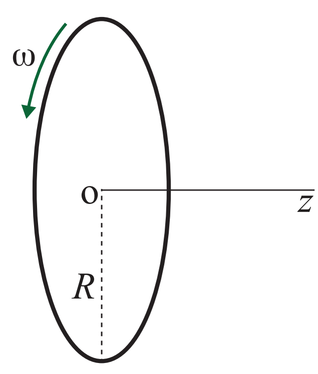

Figure 1: Calculation of magnetic field of a rotating charged disk.

Formula Sheet

$$\int \frac{x^3}{(x^2+a^2)^{3/2}} dx = \frac{2a^2+x^2}{(x^2+a^2)^{1/2}}.$$