Qualifying exam - August 2023

Classical Electrodynamics

You can use one textbook. Please write legibly and show all steps of your derivations. Note the Formula Sheet attached.

Problem 1 [30 points]
A conducting sphere of radius R carrying a charge q is coated with an insulating layer of thickness h (Fig. 1). Assuming that the coating is an isotropic linear dielectric material with a dielectric constant ε_r, find the electric potential inside the sphere ($r \leq R$).

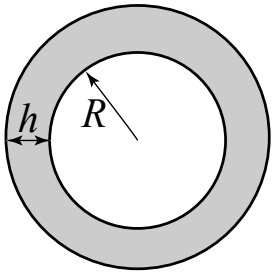

Figure 1: Charged conducting sphere coated with dielectric material.

Problem 2 [30 points]
A point charge q is a distance $d > R$ away from the center of an electrically neutral conducting sphere.

1. Find the charge density on the surface of the sphere. [8 points]
2. Find the force acting on the charge q. [8 points]
3. Find the potential energy of the system. [14 points]

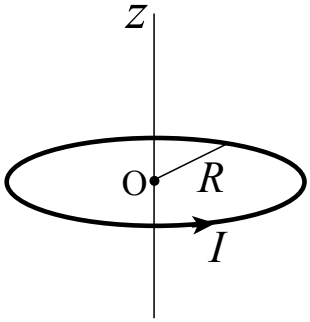

Figure 2: Circular loop of current I.

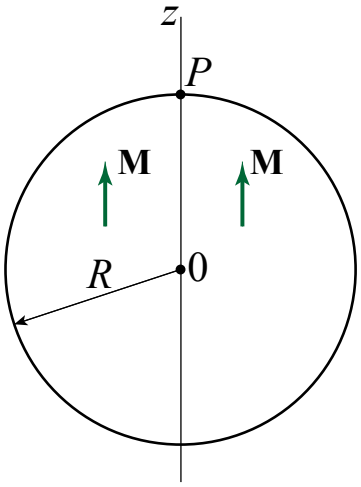

Figure 3: Uniformly magnetized sphere with radius R.

Problem 3 [10 points]

Consider a circular loop of radius R carrying a steady current I. Calculate and sketch the magnetic field on the z-axis as a function of coordinate z. The z-axis is normal to the loop and passes through its center O at $z = 0$ (Fig. 2).

Problem 4 [30 points]

A sphere with radius R has uniform magnetization $\mathbf{M}$ (Fig. 3).

1. Show that the surface bound current density is the same as that generated by a spherical shell of radius R and uniform surface charge σ, rotating around the z axis with a particular angular speed ω. Find the relation among M, R, σ and ω.

2. Calculate the magnetic field at the north pole P of the sphere ($z = R$).

Formula Sheet

$$\int \frac{\sin^3\theta d\theta}{(1-\cos\theta)^{3/2}} = \frac{8(5+\cos\theta)\sin^4(\theta/2)}{3(1-\cos\theta)^{3/2}}.$$