Qualifying exam - August 2021

Electricity and Magnetism

You can use one textbook. Please write legibly and show all steps of your derivations.

Problem 1 [25 points]
Two semi-infinite grounded conducting planes meet at right angles. A point charge q is placed in the region between the planes as shown in Fig. 1.

1. [6 points] Calculate the potential in this region.
2. [9 points] Calculate the force on charge q.
3. [10 points] Find the electrostatic potential energy of the system.

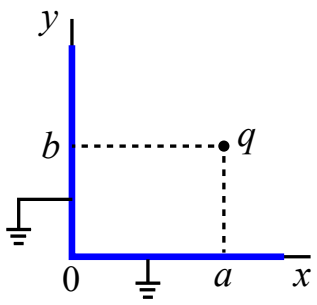

Figure 1: Point charge q in the region between two grounded conducting planes intersecting at right angles.

Problem 2 [35 points]
A grounded conducting sphere of radius R_1 is located inside a concentric non-conducting spherical shell of radius $R_2 > R_1$. The charge density on the shell is fixed at $\sigma(\theta) = \sigma_0(3\cos^2\theta - 1)$, where θ is the polar angle.

1. [20 points] Find the potential inside and outside the charged shell.
2. [15 points] Find the charge density and the total induced charge on the surface of the conducting sphere.

Problem 3 [25 points]
A cable consists of two infinitely long coaxial cylindrical shells of radii R_1 and R_2 ($R_2 > R_1$). The space between the cylinders is filled with an isotropic linear dielectric material with a dielectric constant ε_r. Suppose the cylinders carry uniform charges λ (inner cylinder) and $-\lambda$ (outer cylinder) per unit length.

1. [10 points] Find the capacitance of the cable per unit length.

2. [15 points] Find the electric energy stored in the cable per unit length.

Problem 4 [15 points]
An infinitely long solid cylinder with a radius R and a uniform charge density ρ rotates with an angular frequency ω around its axis. Calculate the magnetic field created by the cylinder.