Classical Electrodynamics Qualifying Exam (3 hours)
August 21, 2019

Solving five of the following six problems (Open-book, closed-notes)

Q1. [20 pts] Two isolated spherical conductors are connected to the positive and negative terminals of a battery with thin wires. Assume vacuum everywhere. Prove that the energy consumed by the battery for charging the conductors is proportional to the square of the voltage of the battery (without assuming the capacitance is known, or using the formula $E=\frac{Q^2}{2C}$).

Q2. [20 pts] A conducting sphere is divided into two non-touching hemispheres by a tiny gap. The hemispheres are charged with Q and -Q respectively. Find the force between the two hemispheres. You can leave the solution in an integral form.

Q3. [20 pts] A point charge q is located in free space a distance *d* from the center of a dielectric sphere of radius *a* ($a < d$) and dielectric constant ϵ/ϵ_0 . Find the potential at all points in space as an expansion in spherical harmonics.

Q4. [20 pts] An electric dipole $\vec{p}$ is placed above an infinitely large grounded conducting plane, at a distance *d*. $\vec{p}$ is parallel to the conducting plane. (1) Find the electric field above the conducting plane; (2) the force on the dipole due to the charges on the conducting plane; (3) the energy stored between the dipole and the conducting plane.

Q5. [20 pts] A conducting sphere with radius *R* is placed in a uniform electrical field E_0 . Knowing that the potential on the sphere is V_0 , find the electric field $\vec{E}$ outside the sphere and the total charge on the sphere.

Q6. [20 pts] A compact circular coil of radius *a,* carrying a current *I* (perhaps *N* turns, each with current *I/N*), lies in the x-y plane with its center at the origin. Find the magnetic induction at any point on the *z* axis.