Qualifying Exam, Electrodynamics, Aug. 2015

1. [30 pts] The scalar potential on a charged thin spherical shell of radius a is given by $\Phi(r = a, \theta) = V \cos(2\theta)$, where θ is the polar angle and V is a constant. Assume vacuum everywhere away from the shell, and $\Phi(r \to \infty) \to 0$.

(a) Determine the scalar potential at the center of the sphere, $\Phi(r = 0)$.

(b) Find $\Phi(r, \theta)$ for $r > a$.

(c) From (b) in the limit of $r \gg a$, deduce the total charge and the electric dipole moment of the charged spherical shell.

2. [20 pts] The lower half space, $z < 0$, is filled with oil with dielectric constant ϵ_1. The upper half space is vacuum with dielectric constant ϵ_0. A dust particle with charge q sits at a distance d above the $z = 0$ plane.

(a) Write down the boundary conditions for the scalar potential at the interface $z = 0$.

(b) Find $\Phi(\rho, \phi, z)$ in cylindrical coordinates by the method of images.

3. [30 pts] An insulating ring of radius a and mass m is suspended in the horizontal plane, and free to rotate about its vertical symmetry axis $\hat{z}$. Total charge Q is uniformly distributed on the ring. A homogeneous external magnetic field B_0 is applied along $\hat{z}$, and the ring is initially at rest. Now the external magnetic field is ramped down to zero. Neglect the effect of self-induction. After the external field becomes zero,

(a) what is the angular velocity of the ring?

(b) what is the magnetic field at the center of the ring?

(c) what is the vector potential $\vec{A}(\mathbf{r})$ for $r \gg a$?

4. [20 pts] A long coaxial cable consists of a thin conducting shell of radius a and a parallel return wire of radius b along the cable axis ($b < a$). Current I is uniformly distributed throughout the cross section of the wire.

(a) Find the magnetic field H as function of ρ, the distance from the cable axis.

(b) Find W, the energy (of the magnetic field) per unit length. Then find L, the self-inductance per unit length, for the cable using the relation $W = LI^2/2$.