

**Classical Mechanics: PhD Qualifying Exam
Open Book (Goldstein only)**

(1) (20 points) Consider a mass M that moves horizontally while attached to a spring of unextended length L and spring constant K . Hanging from this mass M is a pendulum of length l and mass m .

Write down the Lagrangian and equations of motion, and obtain normal modes (both frequencies and eigen vectors) of the system.

(2) (20 points) Consider the motion of a particle of mass m moving in an attractive central-force field described by the central force $f(r) = -k/r^3$.

(a) Obtain the effective potential energy and the equation of the orbit, and discuss the motion for various values of angular momentum. Show that there is a special value of angular momentum where the particle spirals towards the force center.

(b) Show that there are no stable circular orbits in this case.

(3) (20 points) Consider a bead of mass m constrained to move along a hoop of radius a . The hoop is rotating with angular velocity ω about the z -axis.

Write down the Lagrangian and Hamiltonian of the system. Define effective potential energy and determine stable equilibrium points of the system, stating the condition for their stability.

(4) (20 points) A system of two degrees of freedom is described by the Hamiltonian,

$$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2 \quad (1)$$

Here a and b are constants. Show that $F_1 = \frac{p_1 - a q_1}{q_2}$ and $F_2 = q_1 q_2$ are constants of motion. Are there any other independent constants of motion?

(5) (20 points) Assuming ψ and ψ^* as independent field variables, obtain the field equation corresponding to the following Lagrangian density:

$$L = \frac{\hbar^2}{8\pi^2 m} \nabla\psi \cdot \nabla\psi^* + V\psi^*\psi + \frac{\hbar}{4\pi i} (\psi^* \partial_t \psi - \psi \partial_t \psi^*) \quad (2)$$

What are the canonical momenta and the Hamiltonian density of the system?