Classical Mechanics Qualifier Exam (Open Book, January 14, 2025)

9:00 a.m. - 12:00 p.m.

NAME: G-NUMBER:

Important instructions:

- Clearly organize and outline your solution path and solutions.
- In your solutions explain the details of your derivations.
- You can use one textbook (e.g., Goldstein). No other additional course material is allowed.

(1.) A uniform distribution of dust in the Solar System adds to the gravitational attraction of the Sun on a planet an additional force:

$$\mathbf{F} = -mc\mathbf{r}, \tag{1}$$

where m is the mass of the planet, c is a constant proportional to the gravitational constant and the density of the dust, and $\mathbf{r}$ is the radius vector from the Sun to the planet (both considered as points). This additional force is very small compared to the direct Sun-planet gravitational force. Calculate the period and angular velocity for a circular orbit of radius r_0 of the planet in this combined field (i.e., gravitational attraction plus the dust effect).

(25 points)

(2.) Derive the Canonical equations of Hamilton using the Legendre transformation for the Hamiltonian.

(10 points)

(3.) Two point masses, m_1 and m_2 are connected by a spring passing through a hole in a smooth table so that m_2 rests on the table surface and m_1 hangs suspended.

(a) Sketch the problem clearly. Assuming m_1 moves only in a vertical direction (line), what are the generalized coordinates for the system?

(b) Write the Lagrange equations for the system.

(c) Reduce the problem to a single second-order differential equation.

(d) Calculate the first integral of motion and discuss the physical significance of the terms in this this equation.

(35 points)

(4.) In one-dimensional systems it is possible to incorporate frictional effects without introducing the dissipation function.

(a) Find the equations of motion for the Lagrangian

$$L = \exp(\gamma t)\left(\frac{m\dot{q}^2}{2} - \frac{kq^2}{2}\right). \tag{2}$$

How is this system described and what kind of equation is the resulting equation of motion? Are there any constant of motion? Is the total energy constant or not? Why/Why not?

(b) Suppose a point transformation is made of the form

$$s = \exp(\gamma t/2)q. \tag{3}$$

What is the effective Lagrangian in terms of s?

(c) Given the above point transformation, find the equation of motion for s

(30 points)

(100 points in total.)