Classical Mechanics Qualifier Exam (January 9, 2024)

9:00 a.m. - 12:00 p.m.

NAME: G-NUMBER:

Important instructions:

- Clearly organize and outline your solution path and solutions.
- In your solutions explain the details of your derivations.

(1.) Derive the Canonical equations of Hamilton using the Legendre transformation for the Hamiltonian.

(10 points)

(2.) Two point masses, m_1 and m_2 are connected by a spring passing through a hole in a smooth table so that m_2 rests on the table surface and m_1 hangs suspended.

(a) Sketch the problem. Assuming m_1 moves only in a vertical direction (line), what are the generalized coordinates for the system?

(b) Write the Lagrange equations for the system and discuss the physical significance any of them may have.

(c) Reduce the problem to a single second-order differential equation.

(d) Calculate the first integral of motion.

(30 points)

(3.) A point particle moves in space under the influence of a force derivable from a generalized potential U of the form:

$$U(\mathbf{r}, \mathbf{v}) = V(r) + \boldsymbol{\gamma} \cdot \mathbf{L}, \tag{1}$$

where $\mathbf{r}$ is the radius vector from a fixed point, $\mathbf{L}$ is the angular momentum about that point, and $\boldsymbol{\gamma}$ is a fixed vector in space. Find the components of the force on the particle in both (a) Cartesian and (b) spherical polar coordinates, on the the basis of the relationship between Q_j and $U(q, \dot{q})$.

(30 points)

(4.) Consider a particle that describes a circular orbit under the influence of an attractive central force directed toward a point on the circle.

(a) Derive the Lagrangian L in polar coordinates with a radial distance r and azimuthal angle of θ and sketch the problem.

(b) Derive the Lagrangian equations of motion.

(c) Write down the canonical momentum for θ, the equation of motion in θ-direction, and the first integral involving the constant magnitude of the angular momentum, l.

(d) Derive a second order differential equation involving r (and the constant magnitude of the angular momentum) only.

(30 points)

(100 points in total.)