Classical Mechanics Qualifier Exam (Jan 2023)

NAME: G-NUMBER:

Important instructions: In your solutions explain the details of your derivations. Present your solutions in a clean and clear way.

(1.) Two point masses, m_1 and m_2 are connected by a spring passing through a hole in a smooth table so that m_2 rests on the table surface and m_1 hangs suspended.

(a) Sketch the problem. Assuming m_1 moves only in a vertical direction (line), what are the generalized coordinates for the system?

(b) Write the Lagrange equations for the system and discuss the physical significance any of them may have.

(c) Reduce the problem to a single second-order differential equation.

(d) Calculate the first integral of motion.

(40 points)

(2.) A Hamiltonian of one degree of freedom has the form

$$H = \frac{p^2}{2a} - bqp\ \exp(-\alpha t) + \frac{ba}{2}q^2 \exp(-\alpha t) + \frac{kq^2}{2}, \tag{1}$$

where a, b, α, k are constants.
Find a Lagrangian corresponding to this Hamiltonian in terms of q and $\dot{q}$, eliminating p.

(20 points)

(3.) A point particle moves in space under the influence of a force derivable from a generalized potential U of the form:

$$U(\mathbf{r}, \mathbf{v}) = V(r) + \boldsymbol{\gamma} \cdot \mathbf{L}, \tag{2}$$

where $\mathbf{r}$ is the radius vector from a fixed point, $\mathbf{L}$ is the angular momentum about that point, and $\boldsymbol{\gamma}$ is a fixed vector in space. Find the components of the force on the particle in both (a) Cartesian and (b) spherical polar coordinates, on the the basis of the relationship between Q_j and $U(q, \dot{q})$.

(40 points)

(100 points in total.)