Classical Mechanics Qualifier Exam

January 2019

1. Two point masses, m_1 and m_2 are connected by a spring passing through a hole in a smooth table so that m_2 rests on the table surface and m_1 hangs suspended.
    a. Assuming m_1 moves only in a vertical direction (line), what are the generalized coordinates for the system?
    b. Write the Lagrange equations for the system and discuss the physical significance any of them may have.
    c. Reduce the problem to a single second-order differential equation.
    d. Calculate the first integral of motion.

2. Consider a linear symmetrical triatomic molecule. In the equilibrium condition, two atoms of mass m symmetrically located on each side of an atom of mass M. All three atoms are on straight line, the equilibrium distances apart being d. Consider vibrations only along the line of the molecule. The interatomic potential can be approximated by two springs of force constant k joining the three atoms.

    a. Sketch clearly the problem. Label all parts.
    b. Write down the potential and kinetic energies and explain each of them.
    c. Write down the secular equation and determine the eigenvalues. What is the physical meaning of these eigenvalues?
    d. Determine the eigenvectors of the normal modes and discuss each case, clearly sketching the modes.

3. The Hamiltonian for a system has the form
$$H = \frac{1}{2}\left(\frac{1}{q^2} + p^2 q^5\right).$$
    a. Find the equation of motion for q.
    b. Find the canonical transformation that reduces H to the form of a harmonic oscillator. Show that the solution for the transformed variables is such that the equation of motion found in part a is satisfied.

4. Derive the Canonical equations of Hamilton using the Legendre transformation for the Hamiltonian.

(Problems 1,2, and 3 are each 30 points. Problem 4 is 10 points. Total: 100 points)