Classical Mechanics Qualifier Exam (August 15, 2023)

9:00 a.m. - 12:00 p.m.

NAME: G-NUMBER:

Important instructions:

- In your solutions explain the details of your derivations.
- Present your solutions in a clean and clear way.

(1.) Suppose that a particle moved in a plane and the potential of the particle depends only on the distance from the origin: $V(r)$

(a) Write the Lagrange equation for the system.

(b) Calculate the Euler-Lagrange equations and show that the force is directed along the radius vector from the origin to the particle.

(30 points)

(2.) Consider a particle that describes a circular orbit under the influence of an attractive central force directed toward a point on the circle.

(a) Derive the Lagrangian L in polar coordinates with a radial distance r and azimuthal angle of θ and sketch the problem.

(b) Derive the Lagrangian equations of motion.

(c) Write down the canonical momentum for θ, the equation of motion in θ-direction, and the first integral involving the constant magnitude of the angular momentum, l.

(d) Derive a second order differential equation involving r (and the constant magnitude of the angular momentum) only.

(e) Derive the equation of the orbit

$$f(r) = \frac{l}{r^2}\left[\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{l}{mr^2}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right) - \frac{l}{mr}\right] \tag{1}$$

(50 points)

(3.) Derive the Canonical equations of Hamilton using the Legendre transformation for the Hamiltonian.

(20 points)

(100 points in total.)