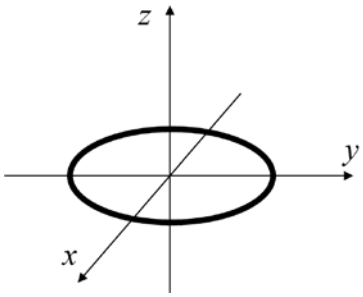
Classical Mechanics Qualifier Fall 2017 George Mason University

There are four questions, and each is worth 20 points.

1. A uniform circular ring of radius R and total mass M lies on the xy plane with its center at the origin.



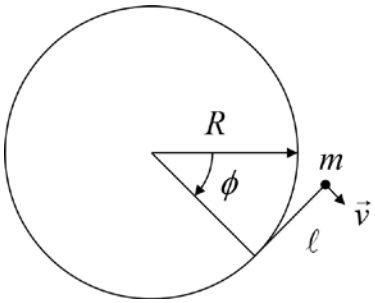
(a) Find the gravitational force $\vec{F}_g$ exerted on a particle of mass m located at a distance z along the z axis.

(b) Find the potential energy of the particle as a function of z assuming that $V \to 0$ when $z \to \infty$.

(c) Find the value of z for which $\left|\vec{F}_g\right|$ is a maximum, and calculate $\left|\vec{F}_g\right|$ at that point.

(d) Show that for $z \ll R$ the motion of the particle is harmonic with time, and find the frequency of oscillation.

2. A point mass m under no external forces is attached to a weightless cord wound around a fixed cylinder of radius R. Initially, the cord is completely wound up so that the mass touches the cylinder at the point shown by the radial arrow. A radially-directed impulse is given to the mass, and the cord unwinds. The magnitude of the mass's initial velocity is v_0, and in the diagram, the length of the unwound cord is labeled ℓ.

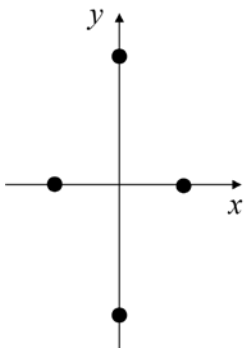


(a) Find the equation of motion in terms of ϕ.

(b) Find the general solution satisfying the initial conditions.

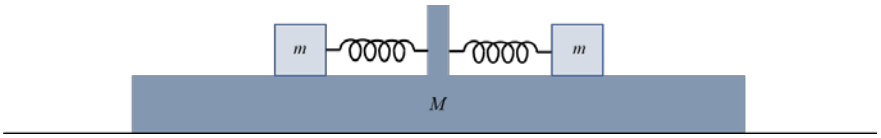
(c) Find the angular momentum of the mass about the cylinder's axis. Explain why your result is either time-dependent or not time-dependent.

3. Four masses, all of value m, lie in the xy plane at positions $(x, y) = (a, 0), (-a, o), (0, +2a)$, and $(0, -2a)$. These are joined by massless rods to form a rigid body.



(a) Find the moment of inertia tensor, using the xyz coordinate system. Write it as a matrix.
(b) Consider a direction given by a unit vector $\hat{n}$ that makes equal acute angles with the $+x$, $+y$, and $+z$ axes. Find the moment of inertia for rotation about this axis.
(c) At a certain time t, the angular velocity vector lies along the above direction $\hat{n}$. Find, for that instant, the angle between the angular momentum vector and $\hat{n}$.

4. A platform of mass M sits on a frictionless surface and is free to move in one dimension. Call that the x direction. Two identical blocks, each of mass m, are connected to a post fixed on the platform by two identical springs of spring constant k. The blocks to move on the platform without friction in the x direction.



(a) Find the normal mode frequencies for the system in terms of m, M, and k.
(b) Give expressions for the normal mode motions of the three masses as functions of time and describe the motions.