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ECE331.

HOMEWORK 3
SOLUTIONS.

1) a)

CD \ AB	00	01	11	10
00	0	1	1	0
01	1	1	1	0
11	0	0	0	0
10	0	1	1	0

$$\overline{X} = \overline{B}\,\overline{D} + A\overline{B} + CD$$

$$\left[\text{SOP } X = B\overline{D} + B\overline{C} + \overline{A}\,\overline{C}D\right]$$

$$\overline{\overline{X}} = \left(\overline{\overline{B}\,\overline{D} + A\overline{B} + CD}\right)$$

$$X = \overline{\overline{B}\,\overline{D}} \cdot \left(\overline{A\overline{B}}\right) \cdot \overline{CD}$$

$$\boxed{X = (B+D)\cdot(\overline{A}+B)\cdot(\overline{C}+\overline{D})}$$

b)

NF \ KM	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

$$\left[\text{SOP: } S = \overline{N} + \overline{F}\right]$$

POS: $\overline{S} = NF$

$$\overline{\overline{S}} = \overline{NF}$$

$$\boxed{S = \overline{N} + \overline{F}}$$

c)

SOP

VT \ UN	00	01	11	10
00	1	1		
01	1	X	X	1
11	X	X		
10	X	X		

SOP:

$$\boxed{J = \overline{U} + \overline{V}T}$$

POS

VT \ UN	00	01	11	10
00	1	1	0	0
01	1	X	X	1
11	X	X	0	0
10	X	X	0	0

POS:

$$\overline{J} = V + U\overline{T}$$

$$\overline{\overline{J}} = \overline{V + U\overline{T}}$$

$$J = \overline{V} \cdot (\overline{U\overline{T}})$$

$$\boxed{J = V \cdot (\overline{U} + T)}$$

2)

a)

$451/2 = 225$ rem. 1 LSB
$225/2 = 112$ rem. 1
$112/2 = 56$ " 0
$56/2 = 28$ " 0
$28/2 = 14$ " 0
$14/2 = 7$ " 0
$7/2 = 3$ " 1
$3/2 = 1$ " 1
$1/2 = 0$ " 1 MSB

$(111000011)_2 = (1C3)_{16}$

$$\boxed{\text{So: } 451_{10} = 111000011_2 = 1C3_{16}}$$

$$\begin{array}{r} 5 \\ 90 \,\overline{|\,16} \\ \underline{80} \\ 10 \end{array}$$

b)

$-90/3 = 30$ rem 0 LSB
$30/3 = 10$ " 0
$10/3 = 3$ " 1
$3/3 = 1$ " 0
$1/3 = 0$ " 1 MSB

10100_3

$90/16 = 5$ rem. 10 → A
$5/16 = 0$ rem 5

$-90_{10} = -5A_{16}$

$$\boxed{\text{So: } -90_{16} = -10100_3 = -5A_{16}.}$$

(these are all unsigned integers!)

c)

$234/7 = 33$ rem. 3
$33/7 = 4$ rem. 5
$4/7 = 0$ rem 4

453_7

$234/6 = 39$ rem. 0
$39/6 = 6$ rem. 3
$6/6 = 1$ 0
$1/6 = 0$ 1

1030_6

$$\begin{array}{r} 39 \\ 6\,\overline{|\,234} \\ \underline{18} \\ 54 \end{array}$$

$$\boxed{\text{So: } 234_{10} = 453_7 = 1030_6}$$

d) $-1F.29_{16}$

-00011111.00101001_2

-037.122_8

$$-1F.29_{16} = -\left[(16^1 + 15) + 2.16^{-1} + 9.16^{-2}\right]$$

$$= -\left[31 + \tfrac{1}{8} + \tfrac{9}{256}\right] = -\left(31.125 + \underbrace{\tfrac{9}{256}}_{\simeq 0.04}\right) = -31.165_{10}$$

To base 6:

- integer part:

$31/6 = 5 \text{ rem } 1$
$5/6 = 0 \text{ rem } 5$ $\Big\}\ 51_6$

- fractional part:

$0.165 \times 6 = 0.990 \rightarrow 0$ (MSB)
$0.990 \times 6 = 5.94 \rightarrow 5$
$0.94 \times 6 = 5.64 \rightarrow 5$
$0.64 \times 6 = 3.84 \rightarrow 3$ LSB

51.0553_6

So: $\boxed{-1F.29_{16} = -37.122_8 = -51.0553_6}$

e) $AAC_{16} = 101001001100_2$

$= 2^{11} + 2^9 + 2^6 + 2^3 + 2^2$

$= 2048 + 512 + 64 + 8 + 4$

$= 2636_{10}$

$$\begin{array}{r} 2048 \\ 512 \\ 64 \\ 12 \\ \hline 2636 \end{array}$$

$$\boxed{AAC_{16} = 101001001100_2 = 2636_{10}}$$

f) $101101001_2 = 169_{16}$

$= (2^8 + 2^6 + 2^5 + 2^3 + 1)_{10}$

$= (256 + 64 + 32 + 8 + 1)_{10} = 361_{10}$

$$\begin{array}{r} 256 \\ 64 \\ 41 \\ \hline 361 \end{array}$$

$$\boxed{101101001_2 = 169_{16} = 361_{10}}$$

g) INTEGER:

$765/2 = 382$	rem	1	LSB
$382/2 = 191$	rem	0	
$191/2 = 95$	rem	1	
$95/2 = 47$		1	
$47/2 = 23$		1	
$23/2 = 11$		1	
$11/2 = 5$		1	
$5/2 = 2$		1	
$2/2 = 1$		0	
$1/2 = 0$		1	MSB

FRACTIONAL:

$0.240 \times 2 = 0.48$	0	MSB
$0.48 \times 2 = 0.96$	0	
$0.96 \times 2 = 1.92$	1	
$0.92 \times 2 = 1.84$	1	
$0.84 \times 2 = 1.68$	1	
$0.68 \times 2 = 1.36$	1	
$0.36 \times 2 = 0.72$	0	
$\vdots$		

$765.240_{10} = 1011111101.0011110_2 = 2FD.3C_{16}$

$(2FD.3C)_{16}$

3) A)

$$\begin{array}{r} 10010 \\ +\,10011 \\ \hline 100101 \end{array}$$

a+b: 100101

$$\begin{array}{r} 10010 \\ \times\,10011 \\ \hline 10010 \\ 10010 \\ 10010000 \\ \hline 101010110 \end{array}$$

a×b: 101010110

or

$$\begin{array}{r} 10011 \\ \times\,10010 \\ \hline 00000 \\ 10011 \\ 00000 \\ 00000 \\ +\;10011 \\ \hline 101010110 \end{array}$$

a×b: 101010110

EXTRA CREDIT:

a-b:

$$\begin{array}{rl} (b) & 10011 \text{ BIG} \\ (a) & -\,10010 \text{ small} \\ \hline & 00001 \end{array}$$

a-b: - 00001

B)

$$\begin{array}{r} -251 \\ -\,63 \\ \hline 334_8 \end{array}$$

a-b = 334_8

$$\begin{array}{r} 251_8 \\ 63_8 \\ \hline 773 \\ 1766 \\ \hline 20653_8 \end{array}$$

EXTRA CREDIT

$$\begin{array}{r} -251 \\ +63 \\ \hline -166 \end{array}$$

$a+b = -166_8$

Obs: IN BASE 8, the sequence of integer numbers is: 0 1 2 3 4 5 6 7 10 11 12 13 14 15 16 17 20 ...

C)

$$\begin{array}{r} A2F_{16} \\ 1C3_{16} \\ \hline BF2_{16} \end{array}$$

a+b: $BF2_{16}$

$$\begin{array}{r} A2F_{16} \\ 1C3_{16} \\ \hline \end{array}$$

TOO HARD. → I'll do it in binary:

$$\begin{array}{r} 1010\,0010\,1111 \\ 0001\,1100\,0011 \\ \hline 1010\,0010\,1111 \\ 1010\,0010\,1111 \\ 1010\,0010\,1111\,0000 \\ 1010\,0010\,1111 \\ 1010\,0010\,1111 \\ \hline \underbrace{1}_{1}\,\underbrace{0001}_{1}\,\underbrace{1111}_{F}\,\underbrace{0000}_{0}\,\underbrace{1100}_{C}\,\underbrace{1101}_{D_{16}} \end{array}$$

$a * b = (11F\emptyset CD)_{16}$

EXTRA CREDIT.

a-b:

$$\begin{array}{r} A2F \\ -\,1C3 \\ \hline 86C_{16} \end{array}$$

Obs- it is also valid if you do all computations in BINARY. (Just not decimal is the requirement on this question).

EXTRA CREDIT:

3) D)

$$\begin{array}{r} 0111101 \\ -\ 101 \\ \hline a+b:\ 0111000_2 \end{array} \qquad \begin{array}{r} 0111101 \\ -\ 101 \\ \hline a\text{-}b:\ -1000010_2 \end{array}$$

$$\begin{array}{r} 0111101 \\ \times\ 101 \\ \hline 0111101 \\ 0000000 \\ 0111101 \\ \hline a\times b = -100110001_2 \end{array}$$

E)

$$\begin{array}{r} 1011.011 \\ +\ 111.101 \\ \hline a+b \quad 10011.000_2 \end{array} \qquad \begin{array}{r} 1011.011 \\ -\ 111.101 \\ \hline 011.110_2 \end{array} (a-b)$$

EXTRA CREDIT

$$\begin{array}{r} 1011.011 \\ \times\ 111.101 \\ \hline 1011011 \\ 0000000 \\ 1011011 \\ 1011011 \\ 1011011 \\ 1011011 \\ \hline a\times b = 1010110.101111_2 \end{array}$$

RADIX POINT