HOMEWORK 2

SOLUTIONS:

① Ⓐ

A	B	C	$\bar{A}+\bar{B}+\bar{C}$	$\overline{ABC}$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

Ⓑ (OLD VERSION OF QUESTION 1b) → see next page.

$$\overline{B + E\bar{S} + T\overline{(G + \bar{M} + U)}}$$

$$\bar{B} \cdot \overline{E\bar{S}} \cdot \overline{T\overline{(G + \bar{M} + U)}} \quad \text{(De Morgan)}$$

$$\bar{B} \cdot (\bar{E} + \bar{\bar{S}}) \cdot (\bar{T} + \overline{\overline{(G + \bar{M} + U)}}) \quad \text{(De Morgan on 2)}$$

$$\bar{B} \cdot (\bar{E} + S) \cdot (\bar{T} + (G + \bar{M} + U)) \quad \text{(Involution ~~complement~~)}$$

$$(\bar{B}\bar{E} + \bar{B}S) \cdot (\bar{T} + G + \bar{M} + U) \quad \text{(Distributive on the first, associative on the second part.)}$$

RESULT: $\bar{B}\bar{E}\bar{T} + \bar{B}\bar{E}G + \bar{B}\bar{E}\bar{M} + \bar{B}\bar{E}U + \bar{B}S\bar{T} + \bar{B}SG + \bar{B}S\bar{M} + \bar{B}SU$ (Distributive)

(Note: question does not ask to minimize.

1 (B) $\overline{\overline{B+E\bar{S}} + T(\overline{G+\bar{M}+U})}$

$(\overline{\overline{B+E\bar{S}}}) \cdot \overline{T(\overline{G+\bar{M}+U})}$ (De Morgan)

$(B+E\bar{S}) \cdot (\bar{T} + (\overline{\overline{G+\bar{M}+U}})$ (~~Complement~~ Involution and De Morgan)

$(B+E\bar{S}) \cdot (\bar{T} + G + \bar{M} + U)$ (~~Complement~~ Involution and Associative)

$B\bar{T} + GB + B\bar{M} + BU + E\bar{S}\bar{T} + E\bar{S}G + E\bar{S}\bar{M} + E\bar{S}U$

(Distributive)

(2)

(a) $X = \bar{A}B\bar{C}\bar{D} + \underline{\bar{A}BCD} + \underline{\bar{A}BC\bar{D}} + A\bar{B}C\bar{D}$

$X = \bar{A}B\bar{C}\bar{D} + \bar{A}BC(D + \bar{D}) + A\bar{B}C\bar{D}$ (Distributive)

$= \bar{A}B\bar{C}\bar{D} + \bar{A}BC \cdot (1) + A\bar{B}C\bar{D}$ (Complement)

$= \bar{A}B\bar{C}\bar{D} + \bar{A}BC + A\bar{B}C\bar{D}$ (Identity)

$= \bar{A}B(C + \bar{C}\bar{D}) + A\bar{B}C\bar{D}$ (Distributive)

$= \bar{A}B(C + \bar{D}) + A\bar{B}C\bar{D}$ (Simplification)

$= \bar{A}BC + \bar{A}B\bar{D} + A\bar{B}C\bar{D}$ (Distributive)

*

(b) $A(A + \bar{A}B) =$

$= A$ (Absorption)

or (better answer!):

$A(A + \bar{A}B) =$

$= A \cdot A + A\bar{A}B$ (Distributive)

$= A + A\bar{A}B$ (Idempotence)

$= A + 0 \cdot B$ (Complement)

$= A + 0$ (Null Element)

$= A$ (Identity)

2 (c) $X = \bar{A}\bar{B}C + \overline{(A+B+\bar{C})} + \bar{A}\bar{B}\bar{C}D$

$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{\bar{C}} + \bar{A}\bar{B}\bar{C}D$ (DeMorgan)

$\bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D$ (Involution)

$\bar{A}\bar{B}C + \bar{A}\bar{B}(C + \bar{C}D)$ (Distributive)

$\bar{A}\bar{B}C + \bar{A}\bar{B}(C + D)$ (Simplification)

$\bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}D$ (Distributive)

$\bar{A}\bar{B}C + \bar{A}\bar{B}D$ (Idempotence)

(d) $X = XYZ[XY + \bar{Z}(YZ + XZ)]$

$= XYZXY + XYZ \cdot \bar{Z} \cdot (YZ + XZ)$ (Distributive)

$= ZXXYY + XY \cdot 0 \cdot (YZ + XZ)$ (Commutative (1st term), Complement (2nd term))

$= ZXY + 0$ (Idempotence and Null Element)

$= XYZ$ (Identity and Commutative)

3 ⓐ $X = \sum(4, 6, 7)$

CD \ AB	00	01	11	10
00		1		
01				
11		1		
10		1		

$$\boxed{X = \bar{A}B\bar{D} + \bar{A}BC}$$

ⓑ $X = \sum 2\text{-}7, 9, 12\text{-}15)$

uv \ gm	00	01	11	10
00		1	1	
01		1	1	1
11	1	1	1	
10	1	1	1	

$$\boxed{X = m + g\bar{u}v + \bar{g}u}$$

ⓒ $F = \sum(0, 1, 4, 5, 7)$

z \ xy	00	01	11	10
0	1			1
1	1		1	1

$$\boxed{F = \bar{Y} + XZ}$$

④

$F = \bar{A}B \oplus (\overline{A+C})$

$F = \overline{\bar{A}B} \oplus \bar{A}\bar{C}$

C \ AB	00	01	11	10
0				1
1	1	1		1

$F = \bar{A}C + A\bar{B}$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

⑤

ⓐ

X3 \ X1X2	00	01	11	10
0	1	1	1	
1		1		

$f = \overline{x_1}x_2 + x_2\overline{x_3} + \overline{x_1}\,\overline{x_3}$

ⓑ

X3 \ X1X2	00	01	11	10
0			1	1
1		1	1	

$f = x_1\overline{x_3} + x_2x_3.$