Electric field modulation of synchronization in neuronal networks

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Abstract

Motivated by the observation that applied electric fields modulate hippocampal seizures, and that seizures may be asynchronous, we modeled synaptically coupled two-compartment hippocampal pyramidal neurons embedded within an electrically resistive lattice in order to examine network synchronization properties under the influence of externally applied electric fields. Excitatory electric fields were shown to synchronize or desynchronize the network depending on the natural frequency mismatch between the neurons. Such frequency mismatch was found to decrease as a function of increasing electric field amplitude. These findings provide testable hypotheses for future seizure control experiments.
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Since the 1950s, it has been generally assumed that seizures are manifestations of excessive synchrony. However, recent experiments using dual intracellular impalements in the CA1 region of rat hippocampal slices suggest that neurons may *desynchronize* as seizures initiate, and resynchronize as seizures terminate. In addition, theoretical work has recently established that sustained activity can be supported by collectively *asynchronous* states [7]. These results, in juxtaposition with recent demonstrations that

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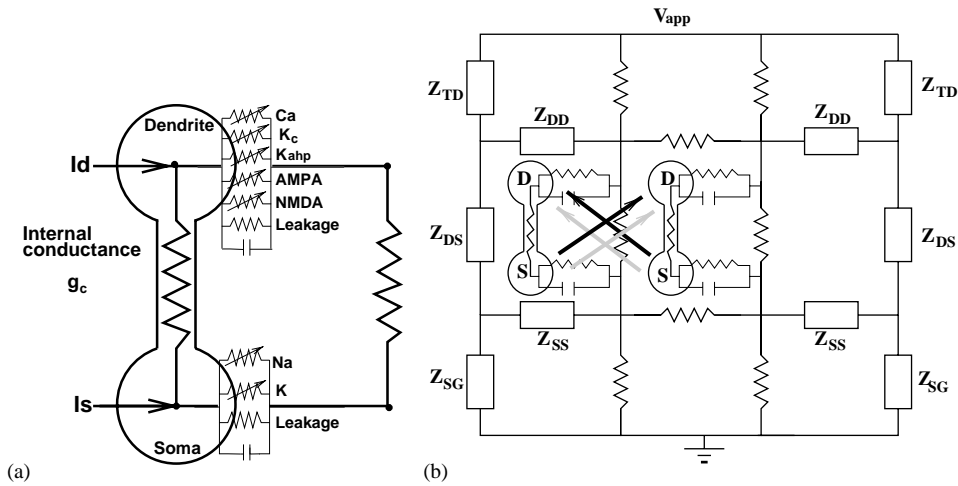*E-mail address:* epark3@gmu.edu (E.-H. Park).

Fig. 1. (a) Currents and conductances for a single PR neuron. (b) Synaptically coupled neurons embedded within a resistive array modeling the electric properties of CA1. Black and gray arrows indicate NMDA and AMPA excitatory synapses, respectively. The terminal resistances are chosen such that the lattice is equivalent to a horizontally infinite lattice.

electric fields can modulate neuronal dynamics and suppress seizure-like activity [4,5], have motivated the present work. We seek to gain a better understanding of the synchronization properties of populations of neurons, and in particular, how such synchrony may be modulated with electric fields.

We report results based on a computational model that we have explicitly designed to include electric field interactions. Since the presence of an electric field induces spatial polarization in neurons [1,2,12], the minimum individual neuronal unit must have at least two spatially separated compartments. We therefore chose the two-compartment lumped Pinsky–Rinzel (PR) model neuron, which consists of a dendritic and a somatic compartment separated by a finite conductance [11]. For the present work, two such neurons were embedded within a lattice of resistors which models the extracellular medium, and a potential difference was imposed along the somatic–dendritic axis. In addition to interacting electrically via the resistive network, the neurons were also coupled via AMPA and NMDA synapses.

A schematic representation of the PR model neuron is shown in Fig. 1(a). Specific details can be found in Refs. [5,11].[1] Two neurons were arranged in the resistive array as shown in Fig. 1(b), and an externally applied electric field was introduced by imposing a potential difference between V_{app} and ground. In addition to varying V_{app}, we also vary the difference Δg_c between the neurons' internal soma-dendrite conductances ($g_c^{(1)} = 2.1$ mS/cm^2, and $g_c^{(2)} = (1 - \delta)g_c^{(1)}$, where $\delta = 0$–30%).

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[1] The parameters were chosen as in Ref. [11], with Id = 0.7 μA/cm^2, Is = 0, $g_{NMDA} = 0.03$ mS/cm^2, $g_{AMPA} = 0.0045$ mS/cm^2 and [K$^+$]$_o$ = 3.5 mM. The potassium reversal potential relative to −60 mV is $V_K = -38.56$ mV.
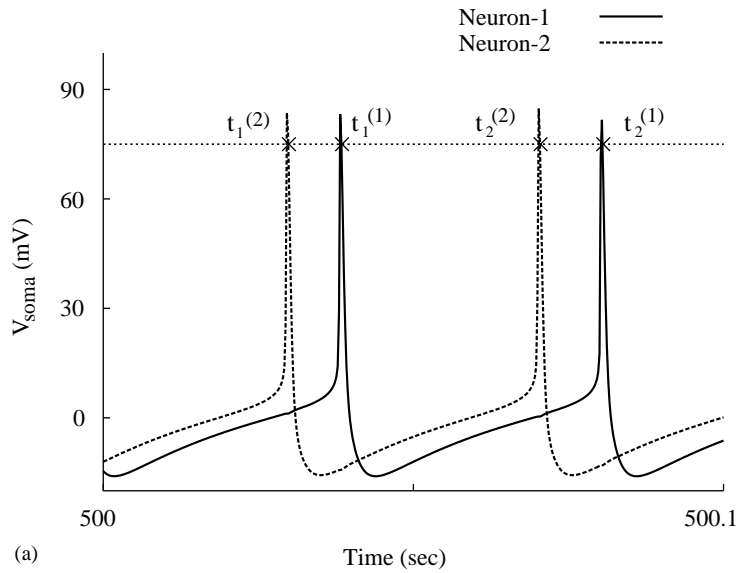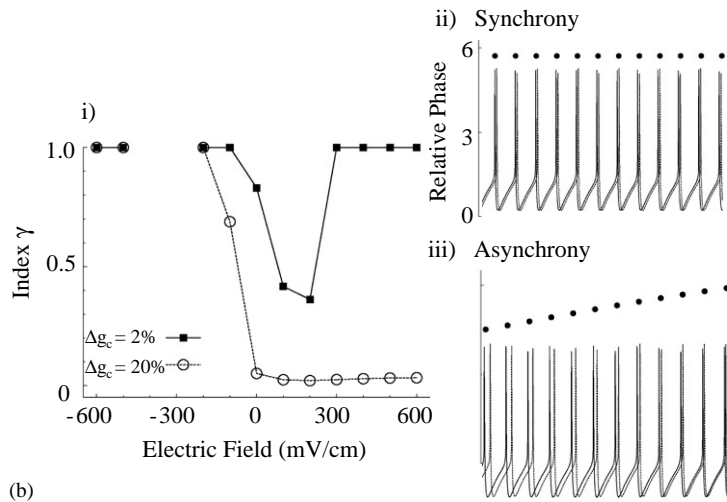
Fig. 2. (a) Tracings of the somatic voltage from two neurons within the network. (b) (i) γ vs. electric field strength for two different levels of parameter mismatch. γ and $V_{\text{soma}}^{(1,2)}$ as a function of time for an electric field of 600 mV/cm in: (ii) the synchronous case, $g_c = 2\%$; (iii) asynchronous case, $g_c = 20\%$.

To characterize synchronization, we employed a phase locking index γ defined by $\gamma^2 = \langle \cos \Psi(t) \rangle^2 + \langle \sin \Psi(t) \rangle^2$ [8,10], where $\langle \ \rangle$ denotes time-average. $\Psi(t)$ is the relative phase, defined as $\Psi(t_k^{(1)}) = 2\pi(t_k^{(1)} - t_m^{(2)})/(t_{m+1}^{(2)} - t_m^{(2)})$, where k and m are event numbers for neurons 1 and 2, respectively, and the times t are defined by threshold crossings. Fig. 2(a) shows sample tracings of the somatic voltages and illustrates the procedure for
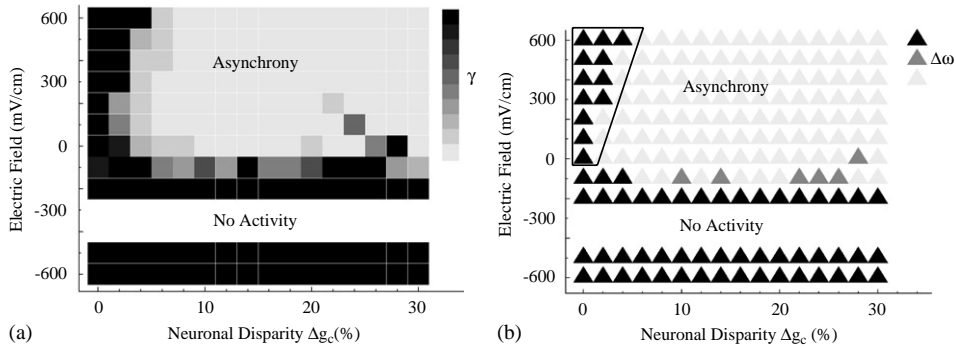
Fig. 3. (a) Synchrony phase diagram. γ is plotted in gray-scale (black = 1, white = 0) as a function of the electric field strength and the neuronal disparity. Network activity is fully suppressed by moderate inhibitory applied fields of approximately -300 to -400 mV/cm. (b) Natural frequency mismatch $\Delta\omega$ between the neurons as a function of the applied electric field and neuronal disparity (black triangle indicates $\Delta\omega < 0.1$; medium gray triangle indicates $\Delta\omega \sim 0.1$; light gray triangle indicates $\Delta\omega > 0.1$).

calculating γ. Note that γ varies between 0 and 1 for unsynchronized and synchronized activity, respectively.

Fig. 2(b) shows that by varying the strength of the applied electric field, the neurons either synchronize or desynchronize depending on the parameter mismatch Δg_c. Neurons with large parameter mismatch are observed to phase-lock for moderate inhibitory (negative) electric fields, and to desynchronize as the field becomes excitatory (positive). This is consistent with results reported by Golomb and Hansel [6], who found that in heterogeneous ensembles, increasing coupling tends to desynchronize network dynamics. However, for small parameter mismatch, we observed that sufficiently large electric fields can resynchronize the neurons.

A more complete summary of our network's dynamics is presented in Fig. 3(a). Here, we show the γ index as a function of both the applied electric field strength and the parameter mismatch Δg_c. Several features are prominent. An extensive asynchronous region is present for large excitatory fields and large neuronal disparity, again, in agreement with Golomb and Hansel's observation in Ref. [6]. Second, large inhibitory fields tend to induce phase locking even when neuronal disparity is large. Third, moderate inhibitory fields suppress collective network activity (consistent with the in vitro seizure suppression experiments of Gluckman et al. [4,5]). Perhaps the most interesting feature of this phase diagram is the boundary between the phase-locked and asynchronous states. As observed in Fig. 2(b), for small neuronal disparity there is a regime where moderately strong excitatory fields desynchronize the network, but stronger fields leads to resynchronization. This threshold is seen in Fig. 3(a) to increase with increasing Δg_c.

We now argue that this nontrivial boundary can be explained using a simple phase oscillator model [3,8,13]. In our network, the dynamics of the individual units are predominately periodic spikes or bursts, and the network is well approximated by a system of coupled phase oscillators for a significant range of parameters. Details
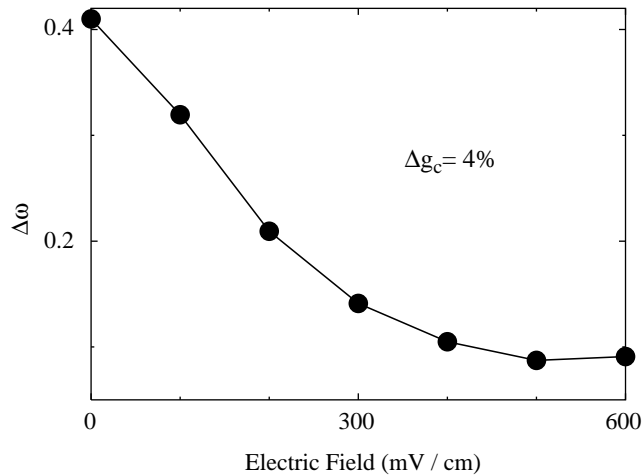
Fig. 4. Natural frequency mismatch $\Delta\omega$ as a function of the applied electric field for $\Delta g_c = 4\%$.

pertaining to the reduction of our system will be presented elsewhere [9]. Here, we show that this simple approach explains the synchrony/asynchrony boundary described above.

Fig. 3(b) shows the degree of natural frequency mismatch $\Delta\omega$ between our two individual units as a function of applied electric field and neuronal disparity. To measure $\Delta\omega$, each neuron was observed in isolation, subject only to the applied electric field. The boundary defined by $\Delta\omega = \alpha \simeq 0.1$ Hz is such that for $\Delta\omega > \alpha$, the network is in the asynchronous state, and for $\Delta\omega < \alpha$, the neurons phase lock to each other. It is remarkable that this simple criterion reproduces the observed transition boundary rather well. In particular, it correctly predicts that for small neuronal disparity ($\Delta g_c < 6\%$), increasing the (excitatory) electric field first desynchronizes and then resynchronizes the network (see the circled region in Fig. 3(b)). To clarify this point, Fig. 4 shows that $\Delta\omega$ decreases for increasing (excitatory) electric field at $\Delta g_c = 4\%$. One can understand this by noting that each individual neuron's spiking rate is a monotonically increasing concave-down function with respect to the applied electric field. Thus, as the spiking rate increases, the corresponding *incremental* increase is smaller. In the small disparity regime, the initial frequency mismatch is small enough such that as the excitatory field increases, the difference between the spiking rates of the two neurons decreases below the critical level $\Delta\omega^* = \alpha$, and phase locking is observed.

We have demonstrated that a computational model of two-compartment neurons embedded within a resistive extracellular medium exhibits both complex phase locking and asynchronous behavior as a function of the applied electric field and the degree of neuronal disparity. The observed dynamics are consistent with recent theoretical and experimental results. Most importantly, there exists a critical value of natural frequency mismatch which determines the synchronization properties of the network. We speculate that the observed phase-locking threshold may be explained in terms of a system of coupled phase oscillators.

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Eun-Hyoung Park received Ph.D. degree in nonlinear dynamics, physics in 2000 Dec. at Potsdam University in Germany. Since March 2001, she is now a post-doc at the Krasnow Institute at George Mason University, Virginia, working on computational modeling, specifically pyramidal cell in hippocampus.

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